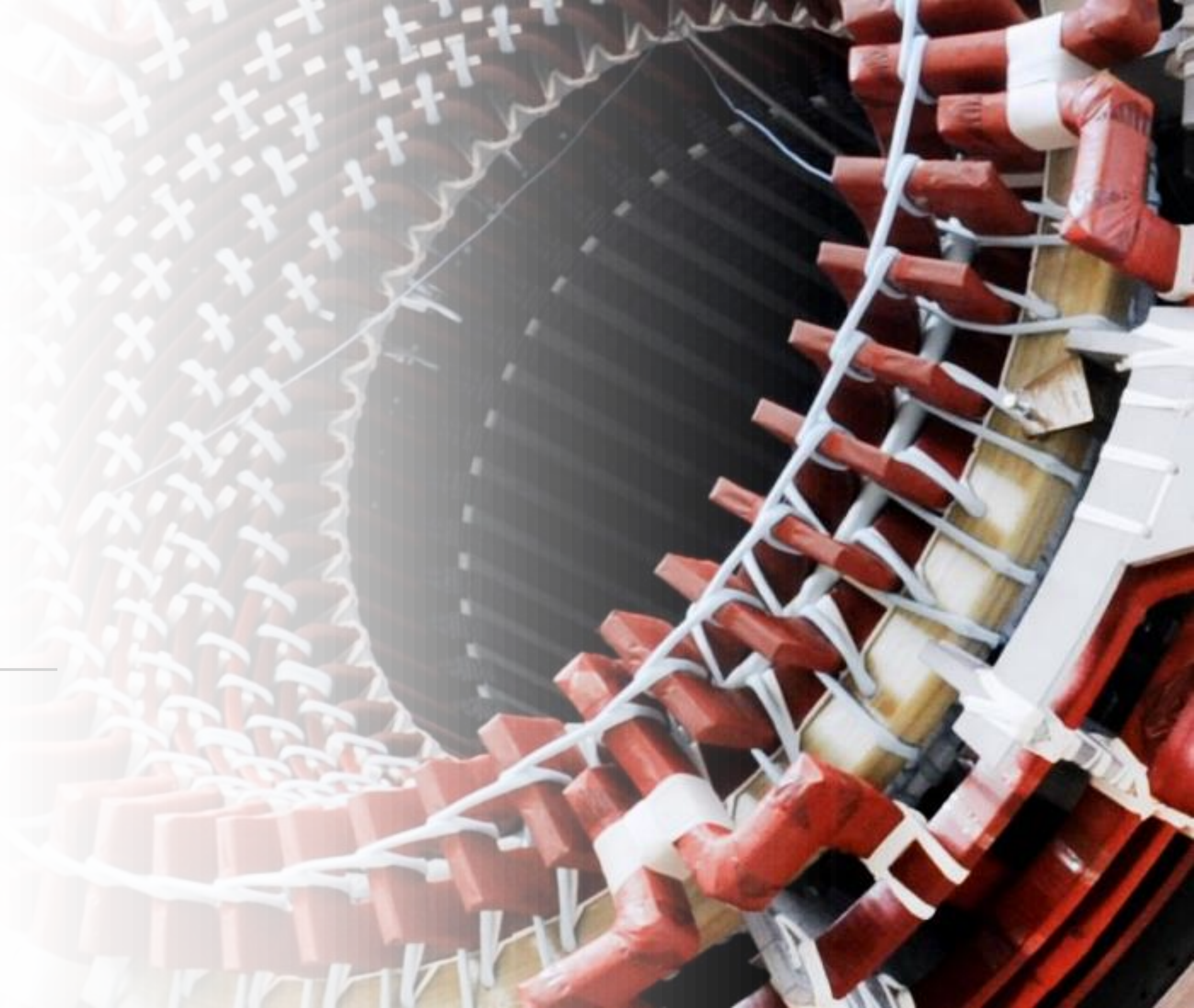
EE3124 Introduction to Electric Machines and Drives

6-Induction Machines



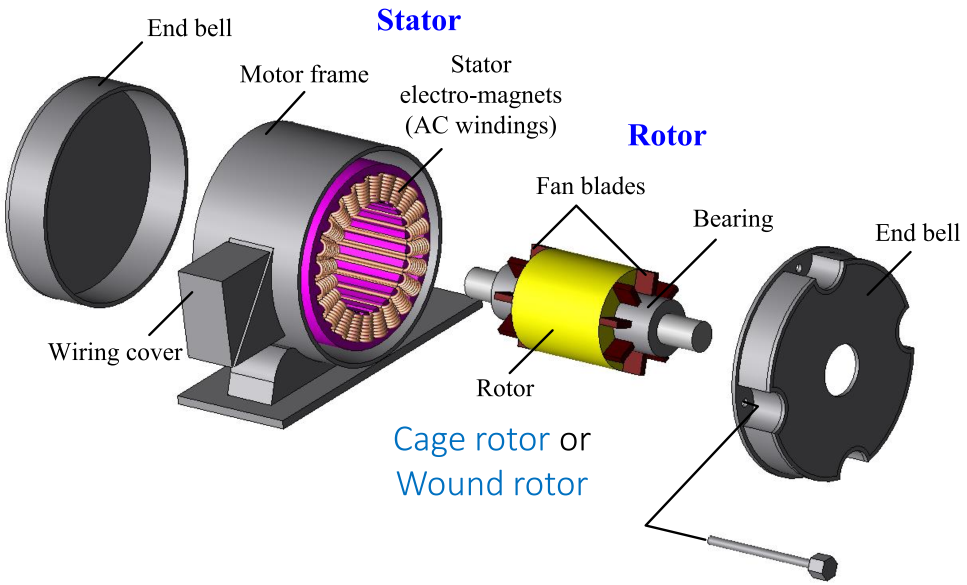
Prof. CQ Jiang

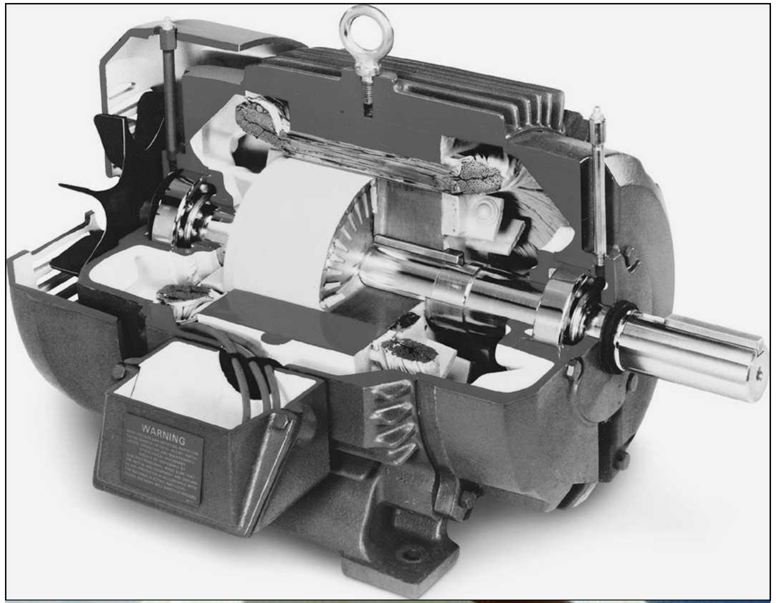
Outline

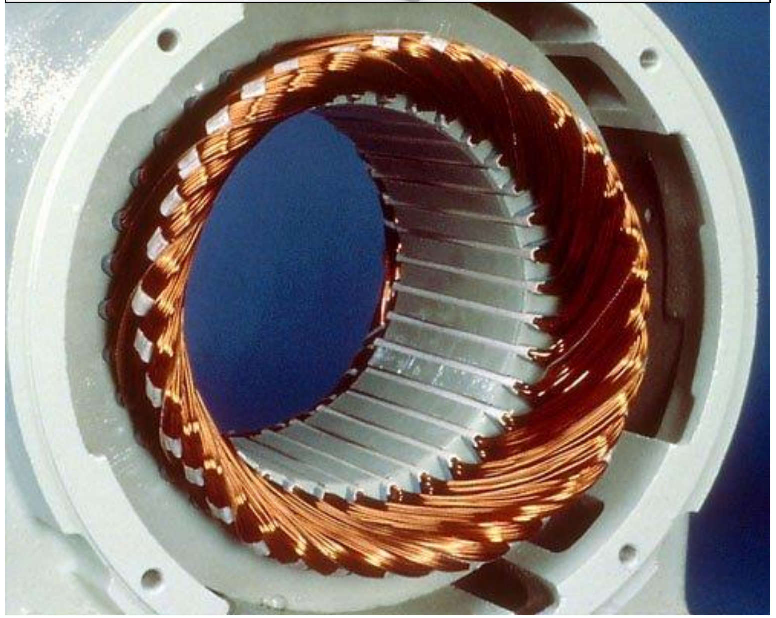
- Induction Motors - Operating Principle
- Electrical Frequency on the Rotor
- The Concept of Rotor Slip
- Final Equivalent Circuit
- Power and Torque in the Motor
- Induction Motor Torque-Speed Characteristics
- Induction Motor Drives and Speed Control

Induction Motors

- They have the same physical stator as a synchronous machine, with a different rotor construction.
- The operation is basically the same as synchronous motors.
- There are two different types of induction motor rotors which can be placed inside the stator.
  - Cage rotor and
  - Wound rotor.

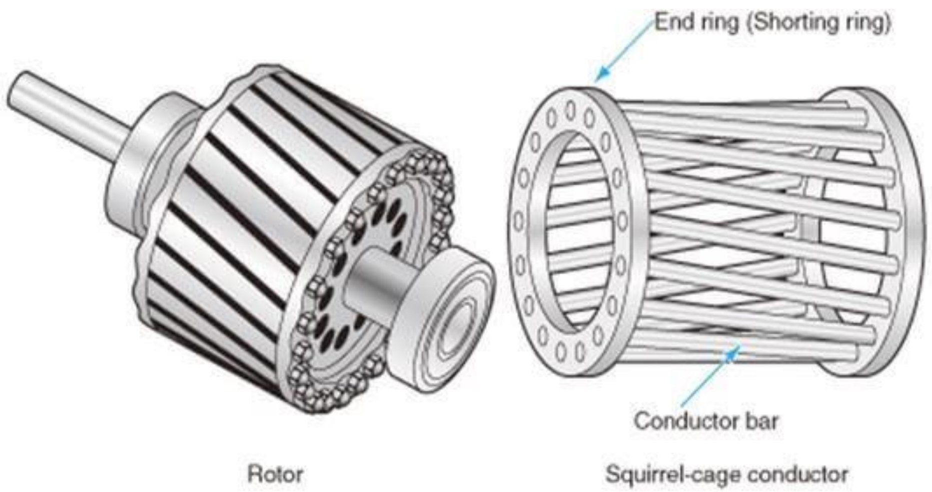


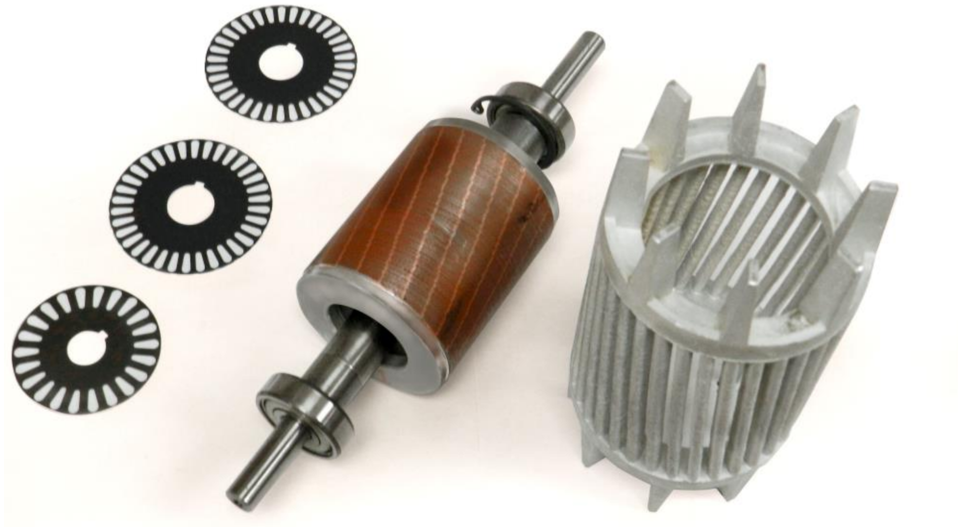


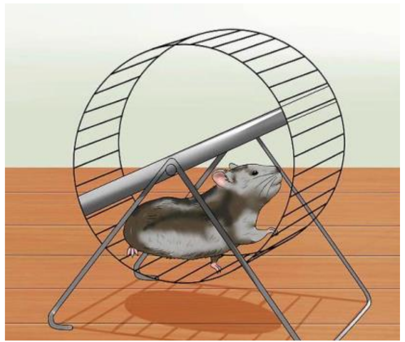


Induction Motors - Cage Rotor

- Rotor consists of a series of conducting bars laid into slots carved in the face of the rotor and shorted at either end by large shorting rings.
- This design referred to as a cage rotor because the conductors.
- It look like on of the exercise wheels that squirrels or hamsters run on.
- Cage rotor induction motors are less expensive than wound induction motor, and they require much less maintenance. As a result, cage rotor induction motors are commonly used

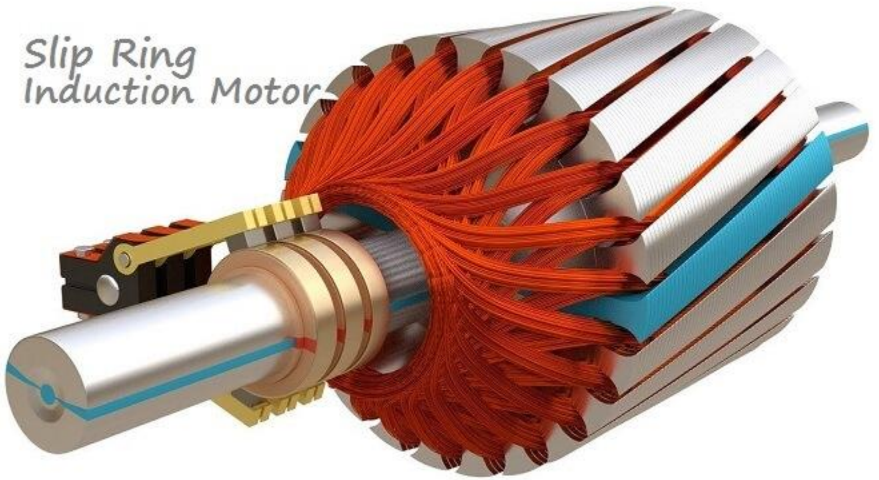


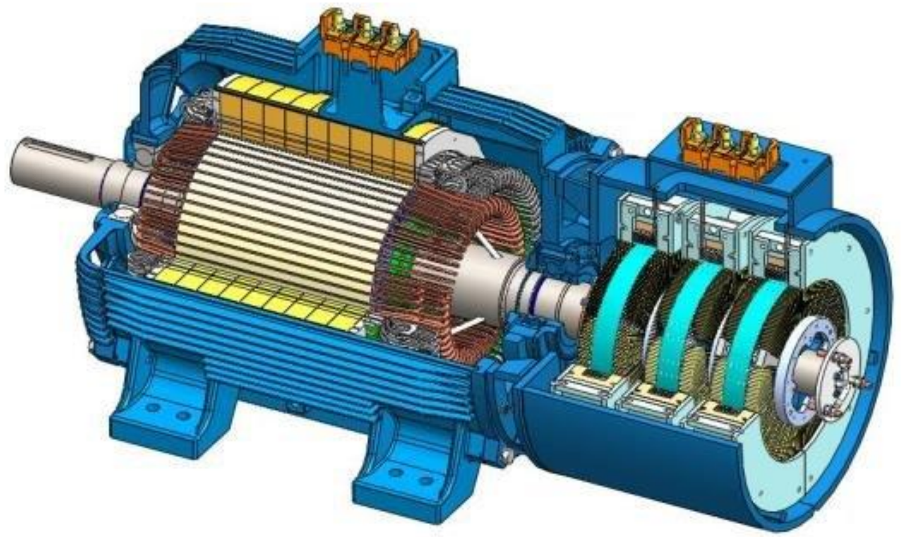




Induction Motors - Wound Rotor

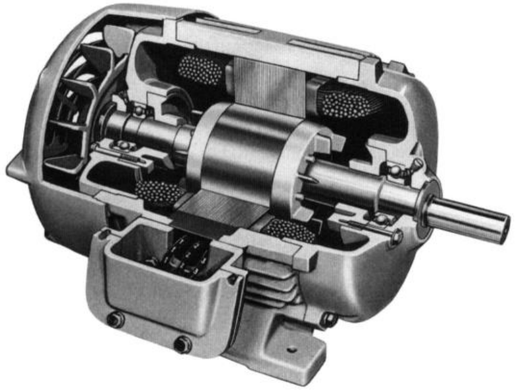
- Rotor has a complete set of three-phase windings that are mirror images of the winding on the stator.
- The three phases of the rotor windings are usually Y-connected and the ends of the three rotor wires are tied to slip rings on the rotor's shaft.
- The rotor windings are shorted through brushes riding on the slip rings. Therefore have their rotor currents accessible at the stator brushes, where they can be examined and where extra resistance can be inserted into the rotor circuit.
- It is possible to take advantage of this feature to modify the torque-speed characteristic of the motor.

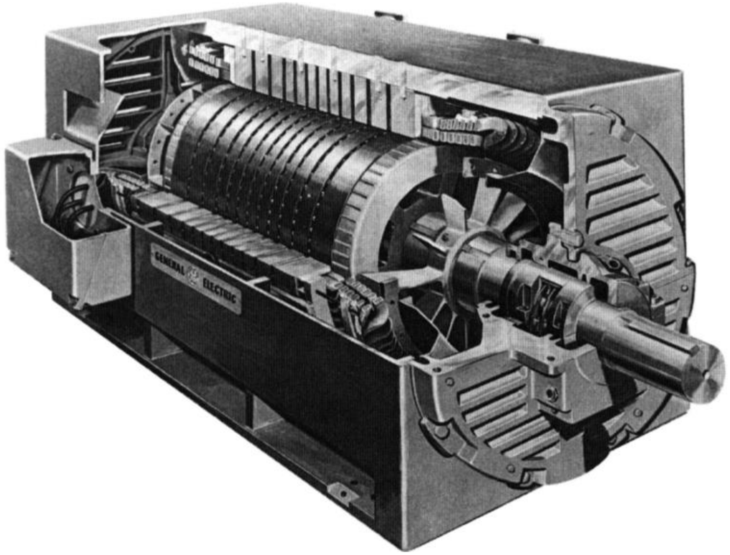




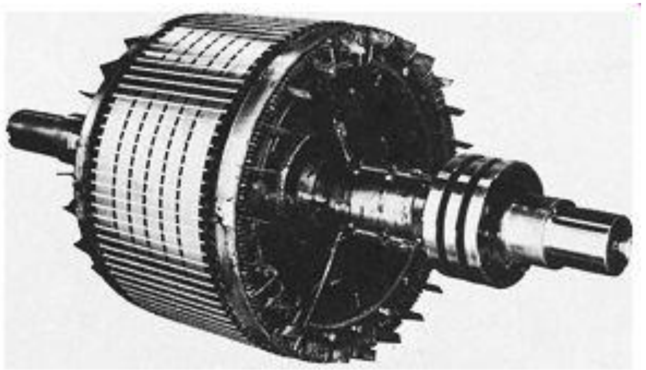
Induction Motors - Cutaway Diagrams

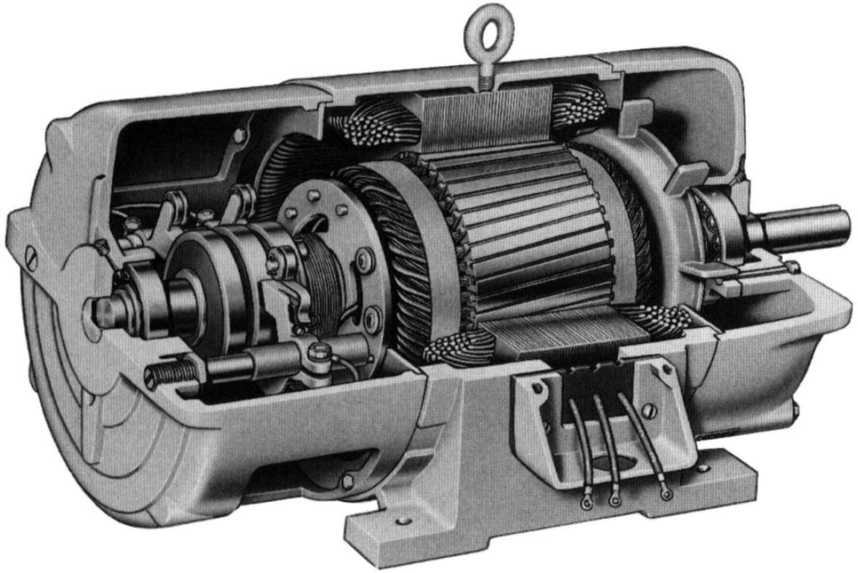
- ❑ Cage Rotor



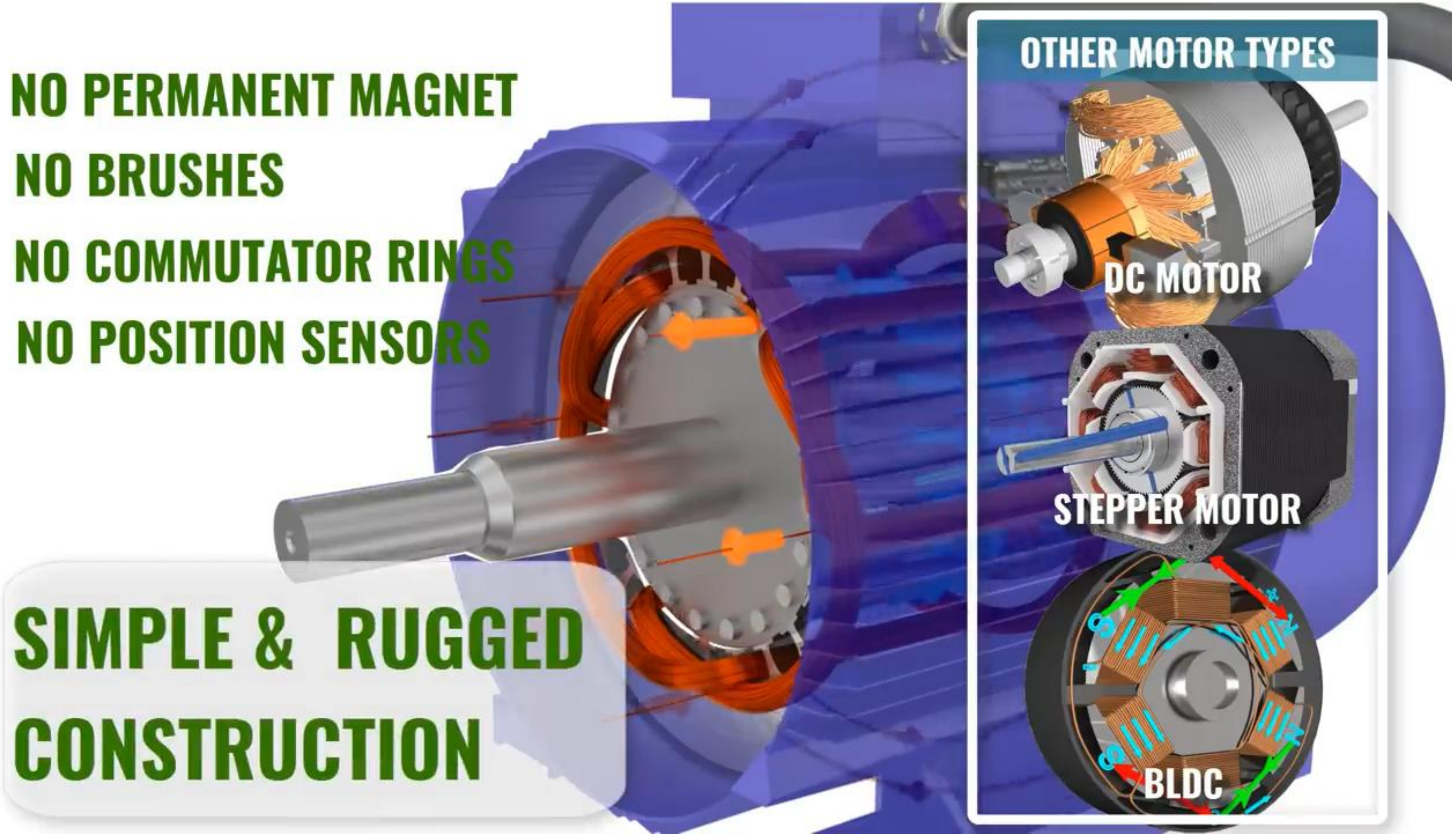


- ❑ Wound Rotor





Induction Motors



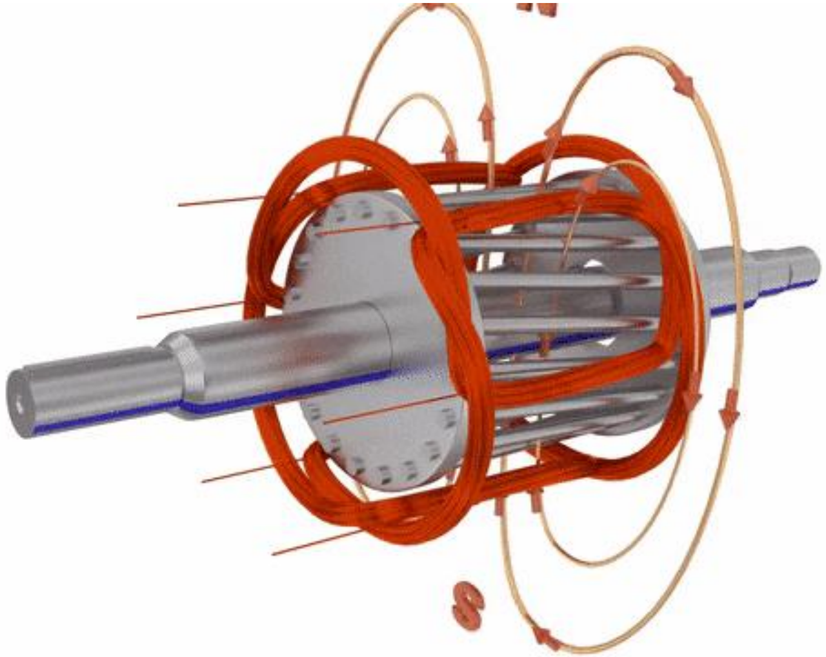
https://www.youtube.com/watch?v=AQqyGNOP_3o

Induction Motors - Operating Principle

❑ Generation of a rotation magnetic field in the stator windings.
(In lecture – AC Machine Fundamentals)

❑ Induced voltage in the rotor windings/conductors.
(In lecture – AC Machine Fundamentals)

❑ Induced torque in the current-carrying loop.
(In lecture – Synchronous Motors)



Induction Motors - Operating Principle

- Generation of a rotation magnetic field in the stator windings.

$$B_{aa'}(t) = B_M \sin \omega t \ \angle 0^\circ \qquad \text{T}$$

$$B_{bb'}(t) = B_M \sin(\omega t - 120^\circ) \ \angle 120^\circ \qquad \text{T}$$

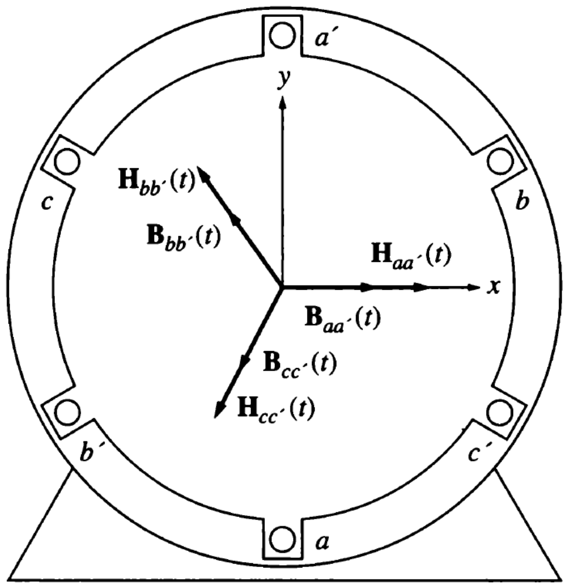
$$B_{cc'}(t) = B_M \sin(\omega t - 240^\circ) \ \angle 240^\circ \qquad \text{T}$$

- The relationship of the electrical frequency and the resulting magnetic field frequency.

$$f_e = \frac{P}{2} f_{sync}$$

- The speed of the magnetic field's rotation.

$$n_{sync}(rpm) = \frac{2 \times f_e}{P} \times 60$$



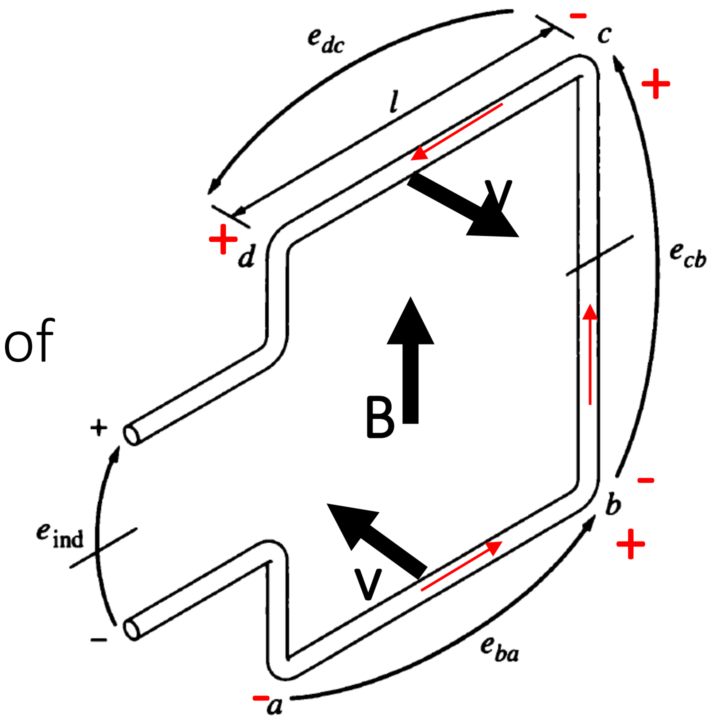
Induction Motors - Operating Principle

- Induced voltage in a conductor

$$e_{ind} = (\mathrm{v} \times \mathrm{B}) \bullet \mathrm{l}$$
$$= (vB \sin 90^\circ) l \cos 0^\circ$$
$$= vBl$$

- Induced voltage in a coils/conductors if it has *Nc* turns of wire/conductor, then

$$e_{ind} = N_c \times 2vB_M l \cos(\omega_m t)$$
$$= N_c \times 2r\omega_M B_M l \cos(\omega_M t)$$
$$= N_c \phi \omega_M \cos(\omega_M t)$$



Induction Motors - Operating Principle

- Induced force in a current-carrying conductor.

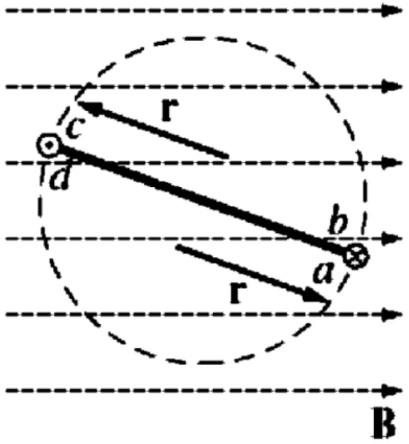
$$F = i(l \times B)$$

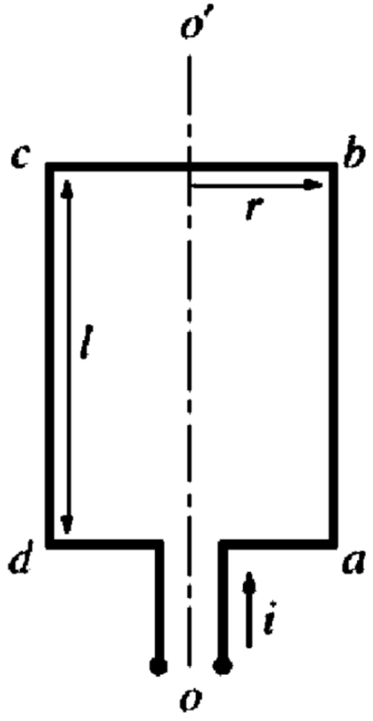
- Induced torque in a current-carrying loop.

$$\tau_{ind} = 2rilB\sin\delta$$

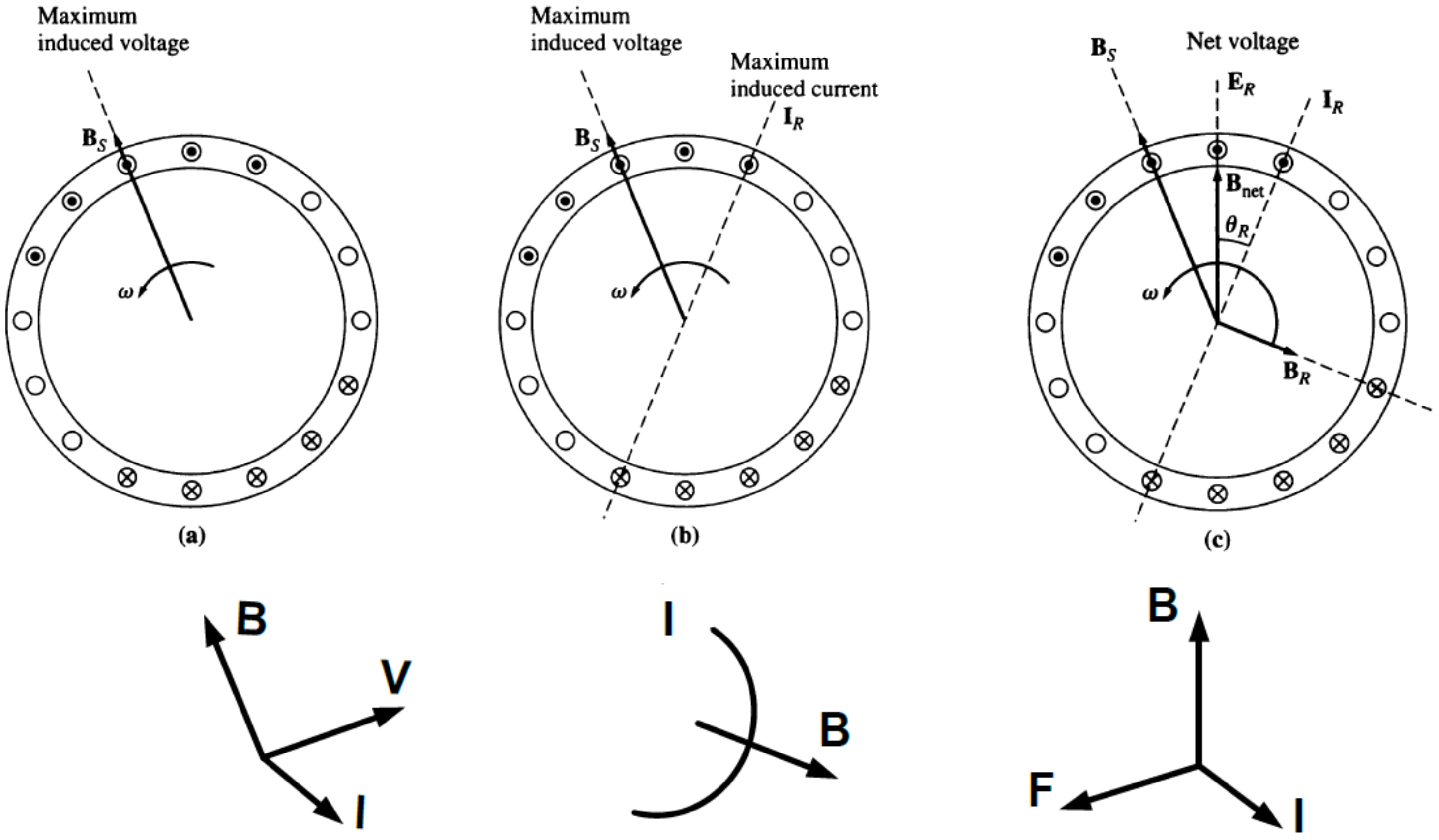
- Induced torque in the machine

$$\tau_{ind} = kB_{loop}B\sin\delta$$





The Development of Induced Torque



The development of induced torque in an induction motor. (a) The rotating stator field *Bs* induces a voltage in the rotor bars; (b) the rotor voltage produces a rotor current flow, which lags behind the voltage because of the inductance of the rotor; (c) the rotor current produces a rotor magnetic field B_R lagging 90° behind itself, and B_R interacts with B_{net} to produce a counterclockwise torque in the machine.

The Development of Induced Torque

- Appling a three-phase set of voltages to the stator winding produce a rotating magnetic field B_S in counterclockwise.

- The velocity of the upper rotor bars relative to the B_S is to the right. Appling right-hand rule, the induced voltage in the upper rotor bars is positive and current flow out of the page.

- Since the rotor is inductive, the rotor current lags behind the rotor voltage.

- The current flow out of the upper bars and into the lower bars. This current flow produces a rotor magnetic field B_R.

- The net magnetic field B_{net} is the sum of the rotor and stator magnetic fields.

The Development of Induced Torque

- B_R interacts with B_S to produce a counterclockwise torque in the machine.

- If the induction motor's rotor were turning at synchronous speed, then the rotor bars would be stationary relative to the magnetic field and there would be no induced voltage. If e_{ind} were equal to 0, then there would be no rotor current and no rotor magnetic field.

- With no rotor magnetic field, the induced torque would be zero, and the rotor would slow down as a result of friction losses.

- An induction motor can thus speed up to near-synchronous speed, but it can never exactly reach synchronous speed.

The Concept of Rotor Slip

- Two terms are commonly used to define the relative motion of the rotor and the magnetic fields.

- **Slip speed**

$$n_{slip} = n_{sync} - n_m$$

- where n_{sync} and n_m are the speed of the magnetic fields and speed of the rotor, respectively.

- **Slip**

$$s = \frac{n_{sync} - n_m}{n_{sync}} \qquad s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}}$$

- If the rotor turns at synchronous speed, s=0, while if the rotor is stationary, s=1. All normal motor speeds fall somewhere between those two limits.

Electrical Frequency on the Rotor

- In induction motor works by inducing voltages and currents in the rotor of the machine, and for that reason it has sometimes been called rotating transformer.

- Like a transformer, the primary (stator) induces a voltage in the secondary (rotor).

- Unlike a transformer, the secondary frequency is not necessarily the same as the primary frequency.

$$f_r = sf_e = \frac{n_{sync} - n_m}{n_{sync}} f_e$$

$$= \frac{n_{sync} - n_m}{\frac{120 f_e}{P}} f_e = \frac{P}{120}\left(n_{sync} - n_m\right)$$

Example 1: Induction Motor Rotation Calculation

- A 208-V 10-hp, four-pole, 60-Hz, Y-connected induction motor has a full-load slip of 5 percent.
  a) What is the synchronous speed of this motor?
  b) What is the rotor speed of this motor at the rated load?
  c) What is the rotor frequency of this motor at the rated load?
  d) What is the shaft torque of this motor at the rated load?

- **Solution**

(a) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_{se}}{P}$$

$$= \frac{120(60 \text{ Hz})}{4 \text{ poles}} = 1800 \text{ r/min}$$

(b) The rotor speed of the motor is given by

$$n_m = (1 - s)n_{sync}$$

$$= (1 - 0.05)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

Example 1: Induction Motor Rotation Calculation

❑ Solution

(c) The rotor frequency of this motor is given by

$$f_{re} = sf_{se} = (0.05)(60\text{ Hz}) = 3\text{ Hz}$$

Alternatively, the frequency can be found

$$f_{re} = \frac{P}{120}(n_{\text{sync}} - n_m)$$

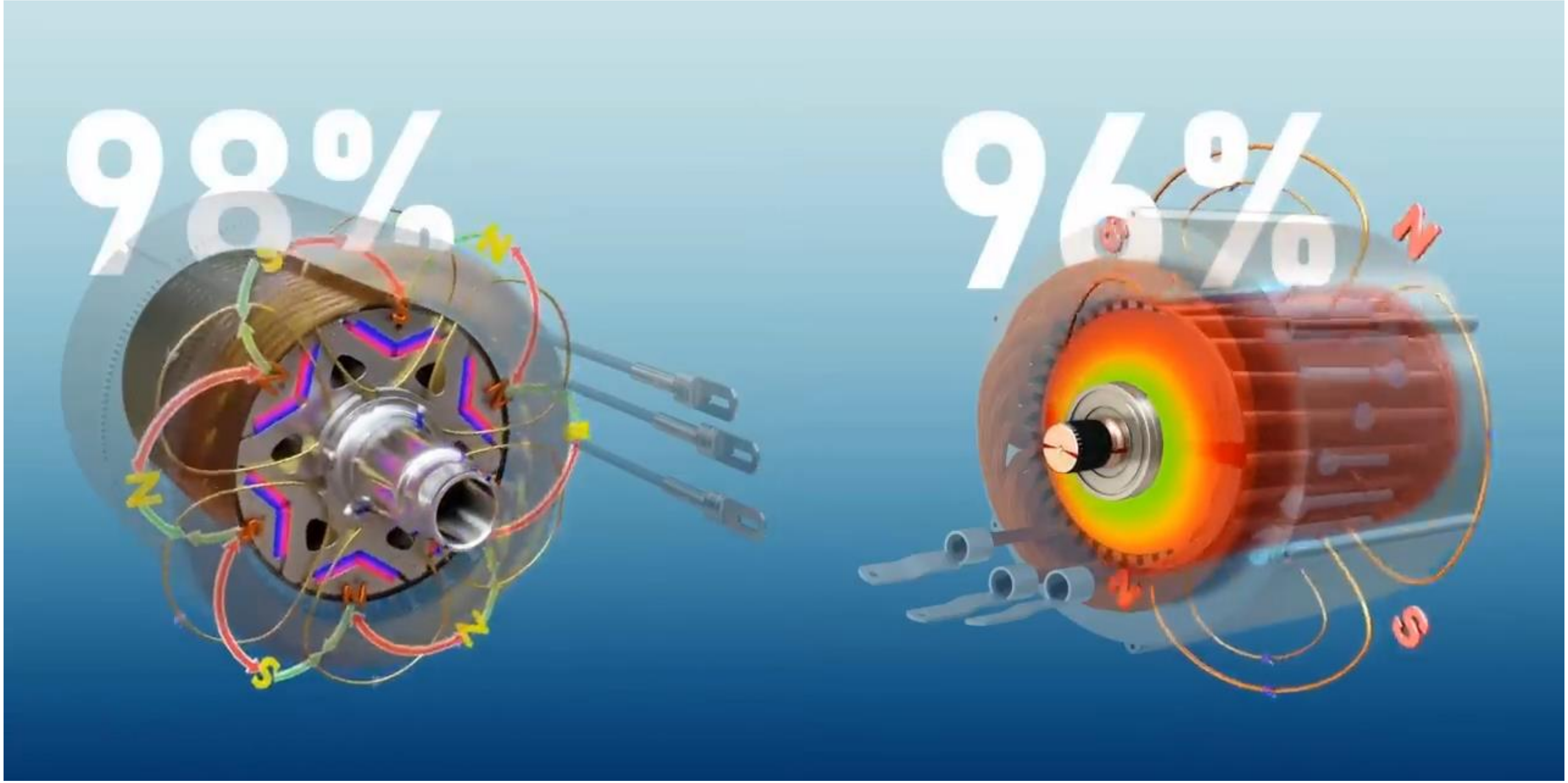
$$= \frac{4}{120}(1800\text{ r/min} - 1710\text{ r/min}) = 3\text{ Hz}$$

(d) The shaft load torque is given by

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m}$$

$$= \frac{(10\text{ hp})(746\text{ W/hp})}{(1710\text{ r/min})(2\pi\text{ rad/r})(1\text{ min/60 s})} = 41.7\text{ N}\bullet\text{m}$$

Comparison between PM synchronous and induction motor



https://www.youtube.com/watch?v=bNgB5z4MWRI&ab_channel=Lesics

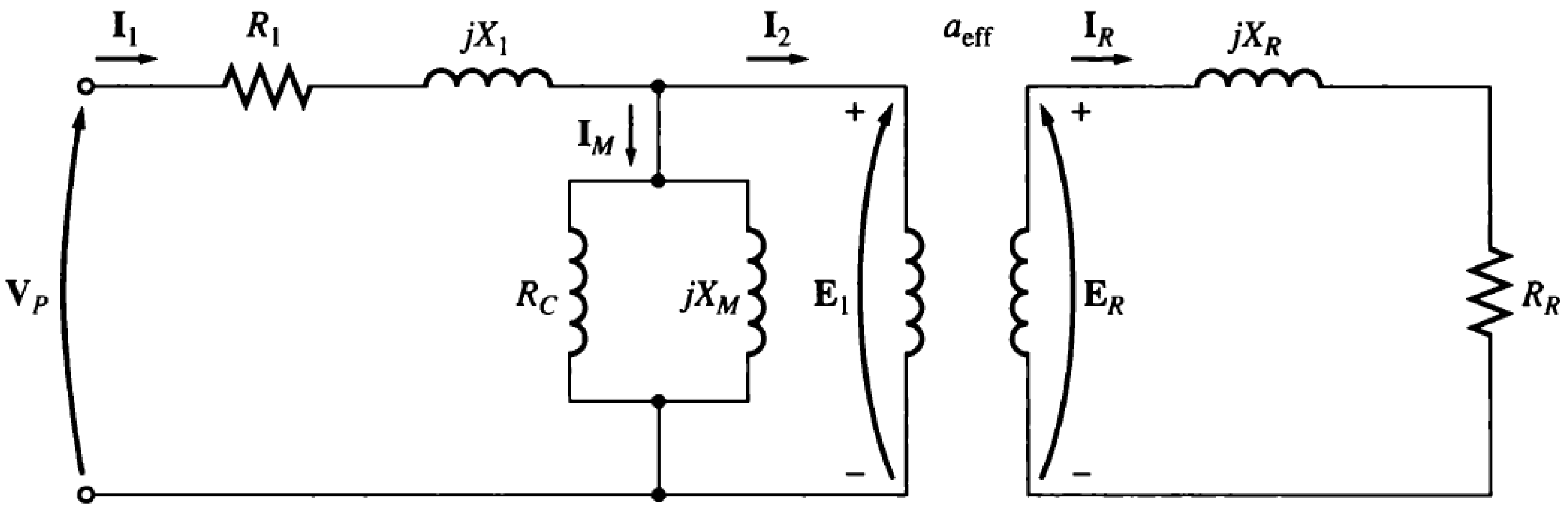
Equivalent Circuit of an Induction Motor

- Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer operation, the equivalent circuit of an induction motor will turn out to be very similar to the equivalent circuit of a transformer.

- However, the air gap in an induction motor greatly increases the reluctance of the flux path, reduces the coupling between primary and secondary windings.

- Thus, higher magnetizing current is required to obtain a given flux. The equivalent magnetizing reactance of an induction motor will have much smaller value than ordinary transformer.

Equivalent Circuit of an Induction Motor



- The transformer model of an induction motor, with rotor and stator connected by an ideal transformer of turns ratio *s*.
- Note: Rotor resistance R_R, ideal transformer of turns ratio $a_{eff.}$
- If core losses are just given by a number (X watts) instead of as a circuit element, they are often lumped together with the mechanical losses and subtracted at the point on the power-flow diagram where the mechanical losses are located.

Equivalent Circuit - Rotor Model

- The induction in the rotor windings depends on the relative motion between the rotor and the stator magnetic fields.
- The largest voltage and rotor frequency are induced in the rotor at stationary (s=1).
- The smallest voltage and frequency occur when the rotor moves at the same speed (s=0) as the stator magnetic field.
- The reactance of an induction motor rotor depends on the inductance of the rotor and the frequency of the voltage and current in the rotor.
- The largest relative motion occurs when the rotor is stationary, called the locked-rotor or blocked-rotor condition. The magnitude of the induced rotor voltage at locked-rotor conditions is called $E_{1,s=1}$.

Equivalent Circuit - Rotor Model

- The magnitude and frequency of the voltage induced in the rotor at any speed is directly proportional to the slip of the rotor

$$f_{re} = sf_{se} \qquad E_R = sE_{R0}$$

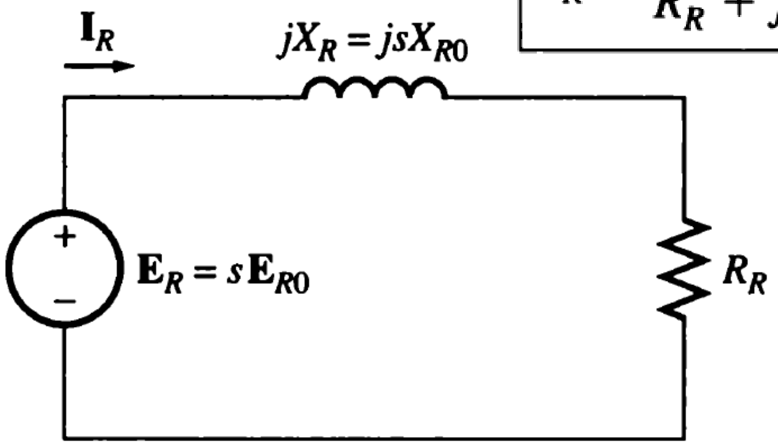
$$X_R = 2\pi s f_{se} L_R = s(2\pi f_{se} L_R) = sX_{R0}$$

where E_{R0}, and X_{R0} are the induced voltage and rotor reactance at stationary condition (blocked-rotor), a rotor inductance of L_R.

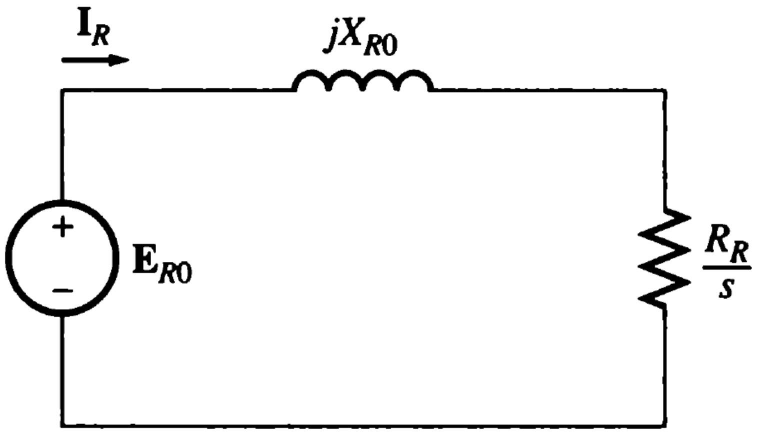
$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jX_R}$$

$$\mathbf{I}_R = \frac{\mathbf{E}_R}{R_R + jsX_{R0}}$$

$$\mathbf{I}_R = \frac{\mathbf{E}_{R0}}{R_R/s + jX_{R0}}$$



The rotor circuit model of an induction motor.



The rotor circuit model with all the frequency (slip) effects concentrated in resistor

Final Equivalent Circuit

- In an ordinary transformer, the voltages, currents, and impedances on the secondary side of the device can be referred to the primary side by means of the turns ratio of the transformer.
- The rotor resistance and the locked-rotor rotor reactance are very difficult or impossible to determine directly on cage rotors, and the effective turns ratio is also difficult to obtain for cage rotors.

$$\mathbf{V}_P = \mathbf{V}'_S = a\mathbf{V}_S$$

$$\mathbf{I}_P = \mathbf{I}'_S = \frac{\mathbf{I}_S}{a}$$

$$Z'_S = a^2 Z_S$$

Primary - Secondary

Ordinary transformer

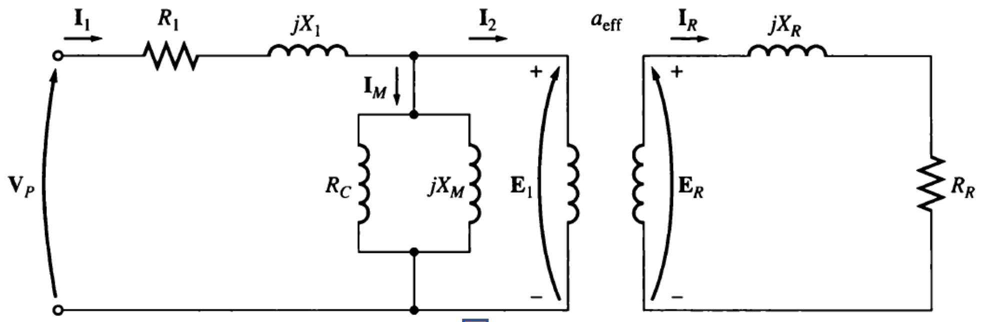
$$\mathbf{I}_2 = \frac{\mathbf{I}_R}{a_{\text{eff}}}$$

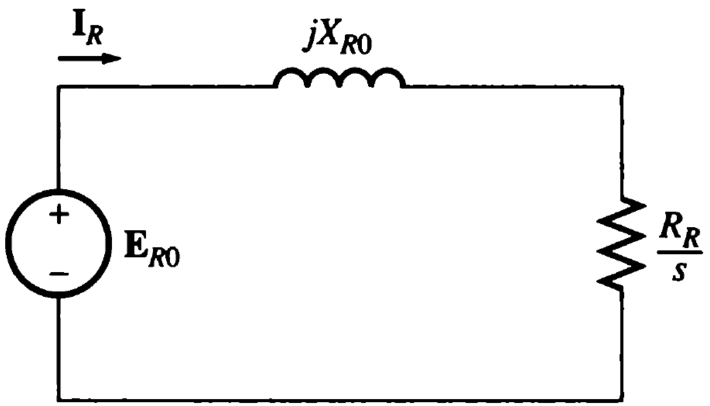
$$Z_2 = a_{\text{eff}}^2\left(\frac{R_R}{s} + jX_{R0}\right)$$

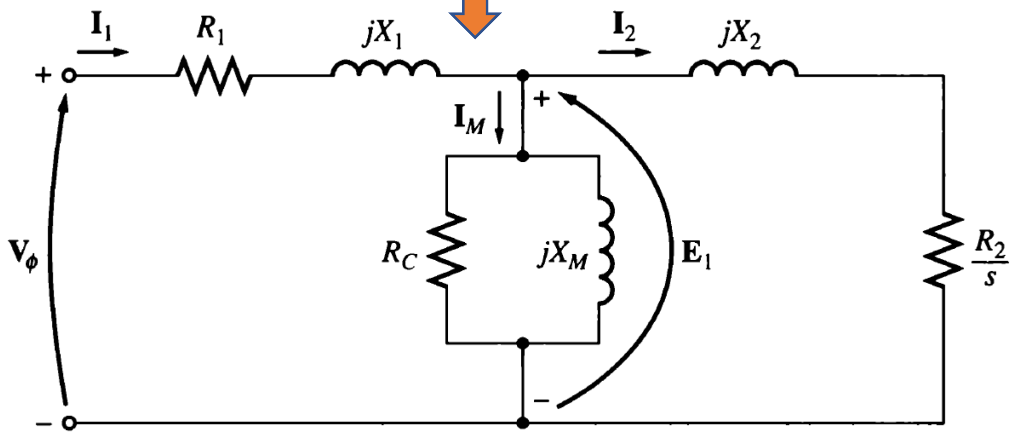
Define:

$$R_2 = a_{\text{eff}}^2 R_R$$

$$X_2 = a_{\text{eff}}^2 X_{R0}$$



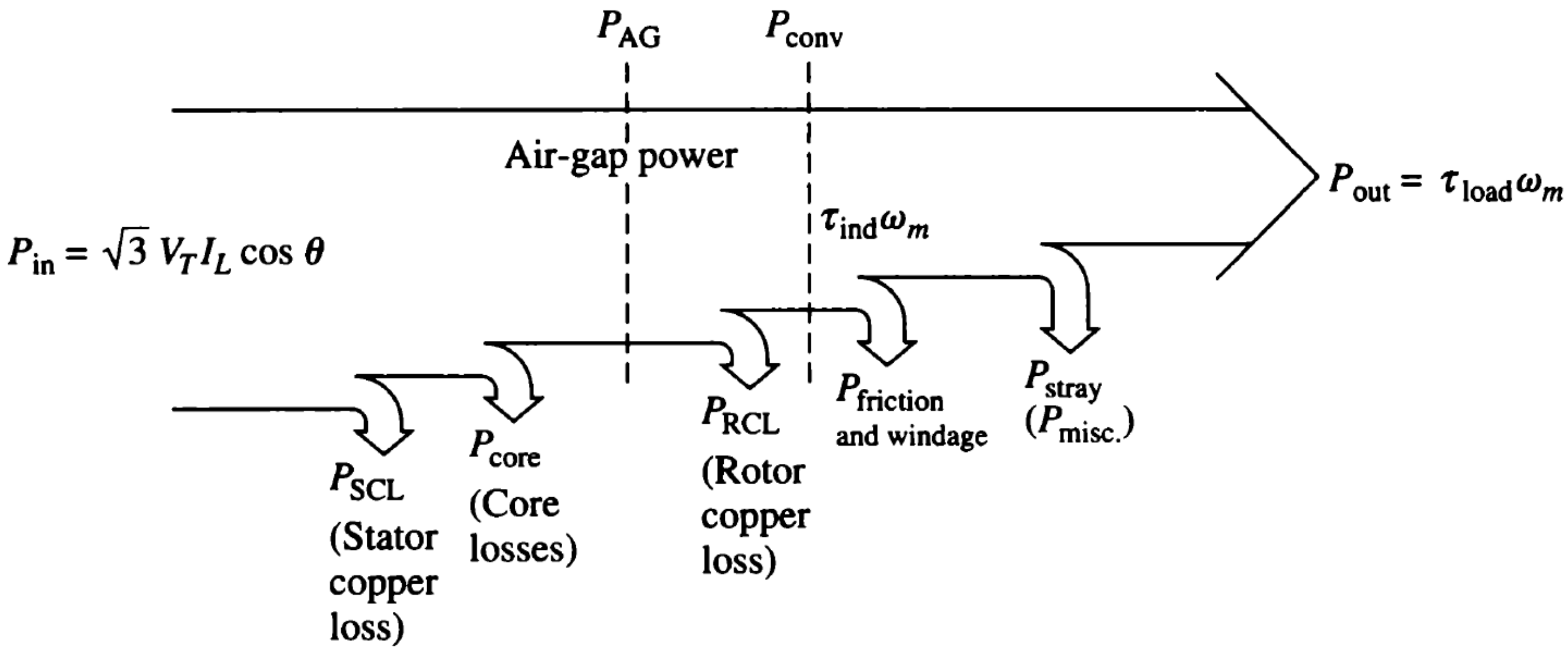




The per-phase equivalent circuit of an induction motor.

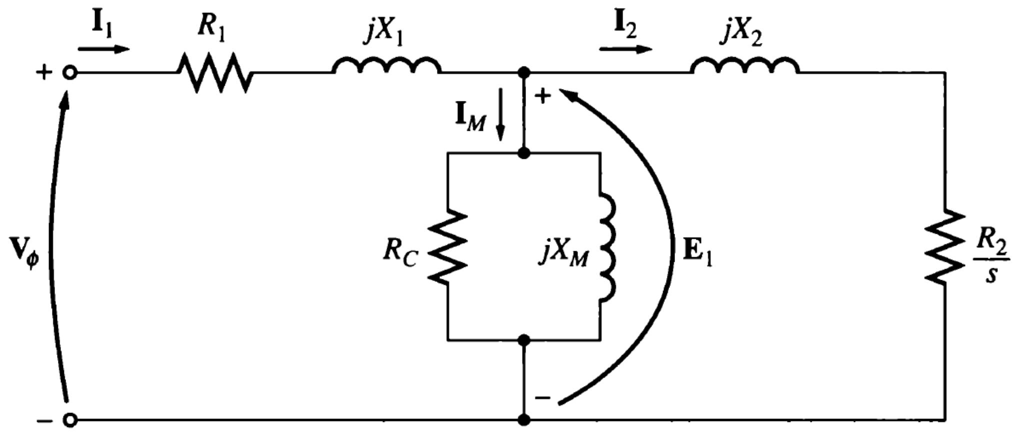
Losses and the Power-Flow Diagram

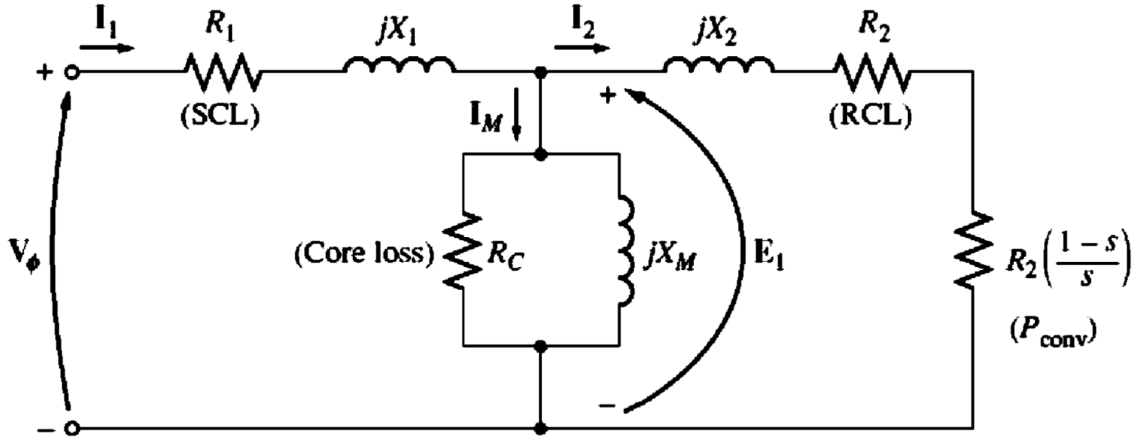
- In an induction motor (the rotor) are shorted out, so no electrical output exists from normal induction motors. Instead, the output is mechanical.
- The higher the speed of an induction motor, the higher its friction, windage, and stray losses.
- On the other hand, the higher the speed of the motor (up to n_{sync}), the lower its core losses.



The power-flow diagram of an induction motor.

Power and Torque in the Motor





$$P_{in} = \sqrt{3} V_{LL} I_{LL} \cos\theta$$

$$P_{SCL} = 3 I_1^2 R_1$$

$$P_{core} = 3 \frac{E_1^2}{R_c}$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$= 3 I_2^2 \frac{R_2}{s} - 3 I_2^2 R_2$$

$$= 3 I_2^2 R_2 \left(\frac{1}{s} - 1 \right)$$

$$P_{conv} = 3 I_2^2 R_2 \left(\frac{1 - s}{s} \right)$$

$$P_{AG} = P_{RCL} + P_{conv}$$

$$= 3 I_2^2 \frac{R_2}{s}$$

$$P_{\text{conv}} = (1 - s) P_{\text{AG}}$$

$$\tau_{ind} = \frac{P_{conv}}{\omega_m}$$

$$= \frac{P_{conv}}{\omega_{sync} - s\,\omega_{sync}}$$

$$= \frac{3 I_2^2 R_2}{s\,\omega_{sync}}$$

Example 2: Power Calculation for Induction Motor

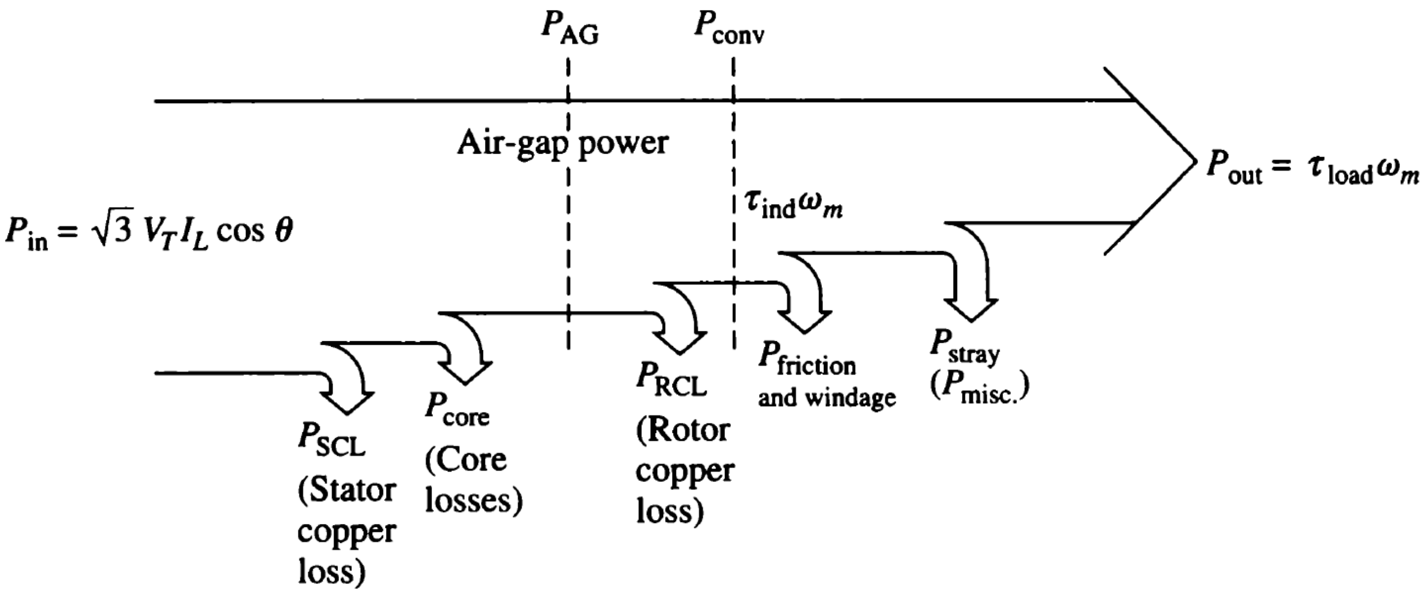
- A 480 V, 60 Hz, 50 hp, three-phase induction motor is drawing 60 A at 0.85 PF lagging. The stator copper losses are 2 kW, and the rotor copper losses are 700 W. The friction is 600 W, the core losses are 1800 W, and the stray losses are negligible. Find the following quantities:

a) The air gap power P_{AG}
b) The power converted P_{conv}
c) The output power P_{out}
d) The efficiency of the motor

Example 2: Power Calculation for Induction Motor

❑ Solution

$$P_{in} = \sqrt{3} V_{LL} I_{LL} \cos\theta$$
$$= \sqrt{3} \times 480 \times 60 \times 0.85$$
$$= 42.4\,\text{kW}$$



$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$
$$= 42.4 - 2 - 1.8$$
$$= 38.6\,\text{kW}$$

$$P_{conv} = P_{AG} - P_{RCL}$$
$$= 38.6 - 700 = 37.9\,\text{kW}$$

$$P_{out} = P_{conv} - P_F$$
$$= 37.9 - 600$$
$$= 37.3\,\text{kW}$$

$$\eta = \frac{P_{out}}{P_{in}}$$
$$= \frac{37.3}{42.4} = 0.88$$

Example 3: Power Calculation for Induction Motor

- A 460 V, 60 Hz, 25 hp, four-pole, Y-connected induction motor has the following impedances in ohms per phase referred to the stator circuit:

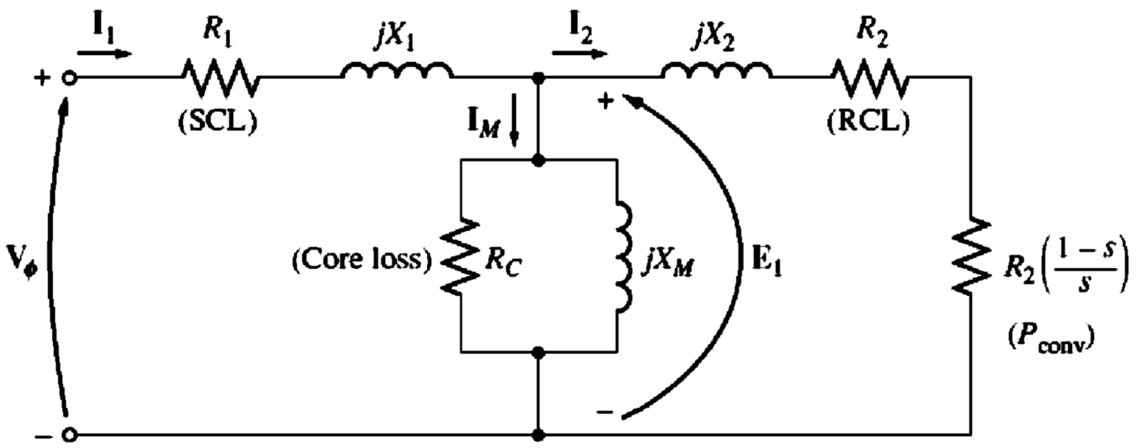
  R_1=0.641, R_2=0.332, X_1=1.106, X_2=0.464, X_M=26.3

  The total rotational losses are 1100W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

  a) Speed
  b) Stator current
  c) Power factor
  d) P_{conv}, P_{load} and τ_{load}
  e) Efficiency

Example 3: Power Calculation for Induction Motor

Solution



(a) The synchronous speed is

$$n_{sync} = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

The rotor's mechanical shaft speed is

$$n_m = (1 - s)n_{sync}$$
$$= (1 - 0.022)(1800\text{ r/min}) = 1760\text{ r/min}$$

(b) To find the stator current, get the equivalent impedance of the circuit. The first step is to combine the referred rotor impedance in parallel with the magnetization branch, and then to add the stator impedance to that combination in series. The referred rotor impedance is

$$Z_2 = \frac{R_2}{s} + jX_2$$
$$= \frac{0.332}{0.022} + j0.464$$
$$= 15.09 + j0.464\ \Omega = 15.10\angle 1.76°\ \Omega$$

$$Z_f = \frac{1}{1/jX_M + 1/Z_2}$$
$$= \frac{1}{-j0.038 + 0.0662\angle -1.76°}$$
$$= \frac{1}{0.0773\angle -31.1°} = 12.94\angle 31.1°\ \Omega$$

$$Z_{tot} = Z_{stat} + Z_f$$
$$= 0.641 + j1.106 + 12.94\angle 31.1°\ \Omega$$
$$= 11.72 + j7.79 = 14.07\angle 33.6°\ \Omega$$

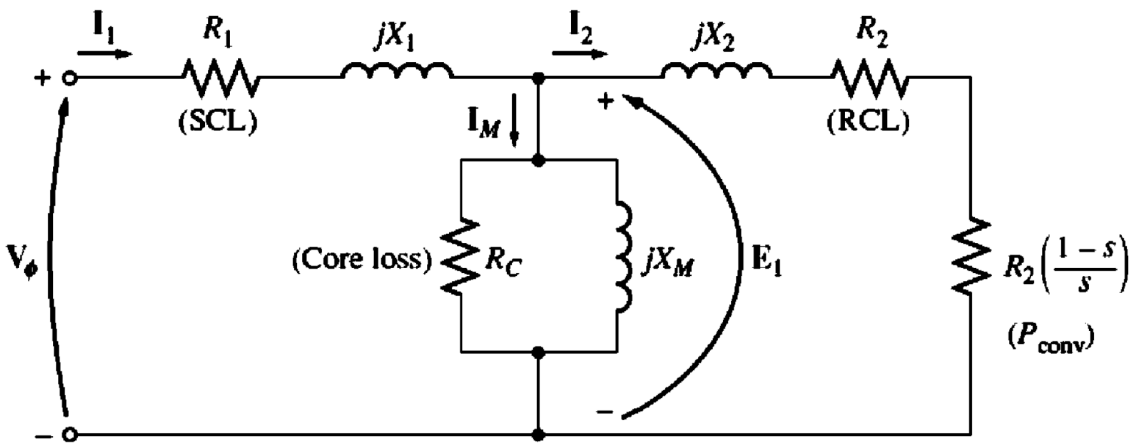
The resulting stator current is

$$\mathbf{I}_1 = \frac{\mathbf{V}_\phi}{Z_{tot}}$$
$$= \frac{266\angle 0°\text{ V}}{14.07\angle 33.6°\ \Omega} = 18.88\angle -33.6°\text{ A}$$

Resistor R_C represents core losses, but will not join any calculations.

Example 3: Power Calculation for Induction Motor

❑ Solution



(c) The power motor power factor is

$$PF = \cos 33.6° = 0.833 \quad \text{lagging}$$

(d) The input power to this motor is

$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$
$$= \sqrt{3}(460\text{ V})(18.88\text{ A})(0.833) = 12{,}530\text{ W}$$

The stator copper losses in this machine are

$$P_{SCL} = 3I_1^2 R_1$$
$$= 3(18.88\text{ A})^2(0.641\ \Omega) = 685\text{ W}$$

The air-gap power is given by

$$P_{AG} = P_{in} - P_{SCL} = 12{,}530\text{ W} - 685\text{ W} = 11{,}845\text{ W}$$

Therefore, the power converted is

$$P_{conv} = (1 - s)P_{AG} = (1 - 0.022)(11{,}845\text{ W}) = 11{,}585\text{ W}$$

The power Pout is given by

$$P_{out} = P_{conv} - P_{rot} = 11{,}585\text{ W} - 1100\text{ W} = 10{,}485\text{ W}$$
$$= 10{,}485\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = 14.1\text{ hp}$$

(e) The induced torque is given by

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$
$$= \frac{11{,}845\text{ W}}{188.5\text{ rad/s}} = 62.8\text{ N}\bullet\text{m}$$

and the output torque is given by

$$\tau_{load} = \frac{P_{out}}{\omega_m}$$
$$= \frac{10{,}485\text{ W}}{184.4\text{ rad/s}} = 56.9\text{ N}\bullet\text{m}$$

(f) The motor's efficiency is

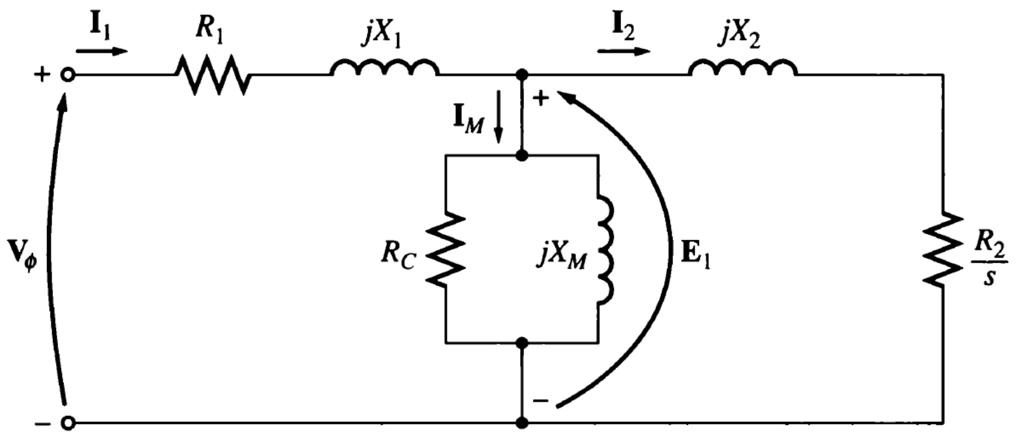
$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$
$$= \frac{10{,}485\text{ W}}{12{,}530\text{ W}} \times 100\% = 83.7\%$$

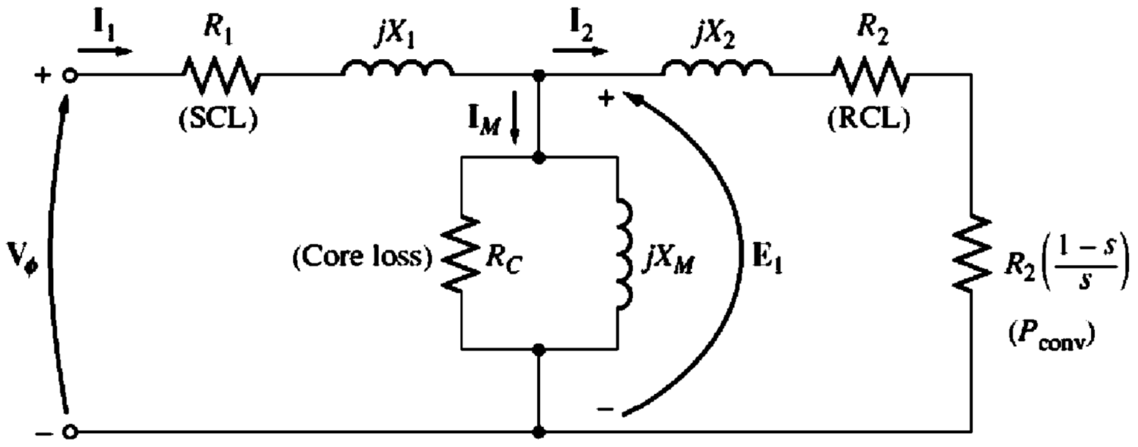
Induction Motor Torque-Speed Characteristics

- Refer to the equivalent circuit, it is possible to derive a general expression for induced torque as a function of speed.

- The induced torque in an induction motor is given by $\tau_{\text{ind}} = \dfrac{P_{\text{conv}}}{\omega_m}$ or $\tau_{\text{ind}} = \dfrac{P_{\text{AG}}}{\omega_{\text{sync}}}$

- Air-gap power P_{AG} is the power crossing the gap from the stator circuit to the rotor circuit. It is equal to the power absorbed in the resistance R_2/s.



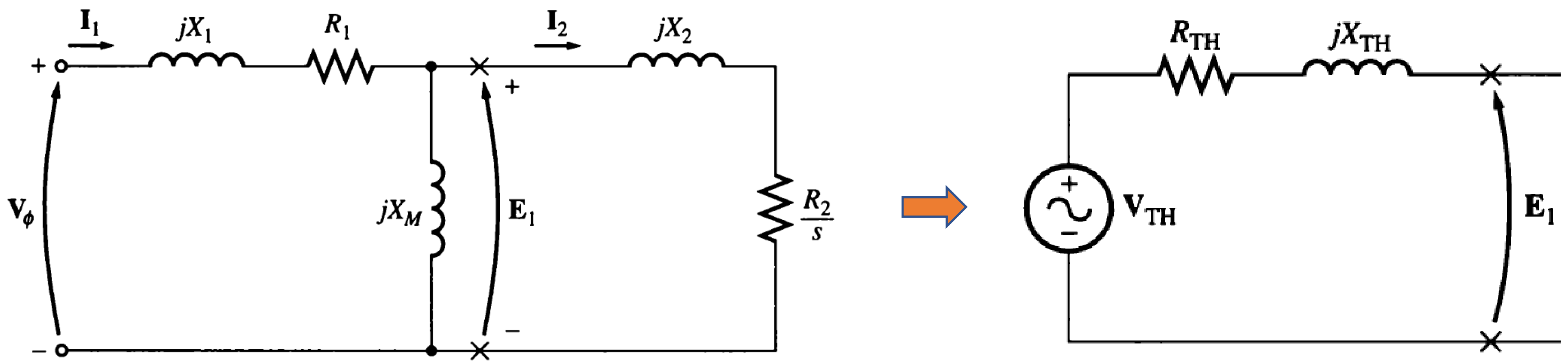


Induction Motor Torque-Speed Characteristics

- The total air gap power is

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

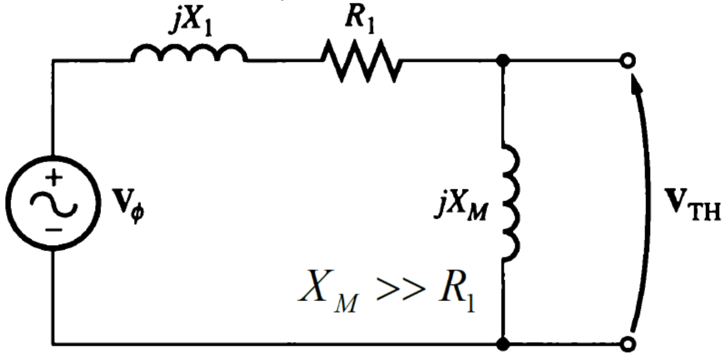
- If I_2 can be determined, then the air gap power and the induced torque will be known.
- Thevenin's equivalent is one of the ways to solve the circuit.
- Thevenin's theorem states that any linear circuit that can be separated by two terminals from the rest of the system can be replaced by a single voltage source in series with an equivalent impedance.



Induction Motor Torque-Speed Characteristics

- Thevenin’s equivalent:
- First, open-circuit the terminals get the Thevenin’s voltage.
- Then, short-circuit the voltage source get the Thevenin's resistance.

Open-circuit



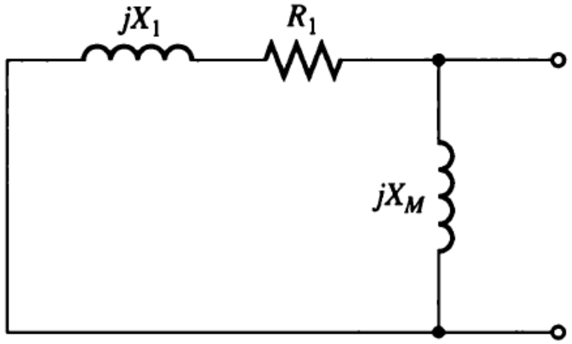
$X_M >> R_1$

$$V_{TH} = V_\phi \frac{jX_M}{R_1 + jX_1 + jX_M}$$

$$= V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

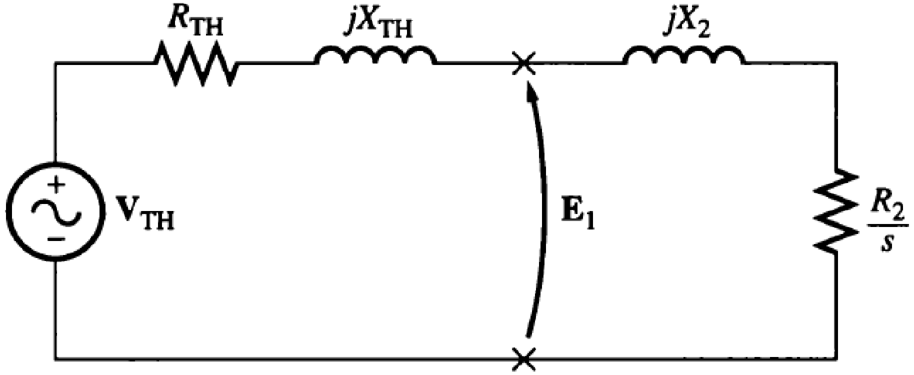
$$\approx V_\phi \frac{X_M}{X_1 + X_M}$$

Short-circuit



$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

Thevenin’s equivalent



$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2$$

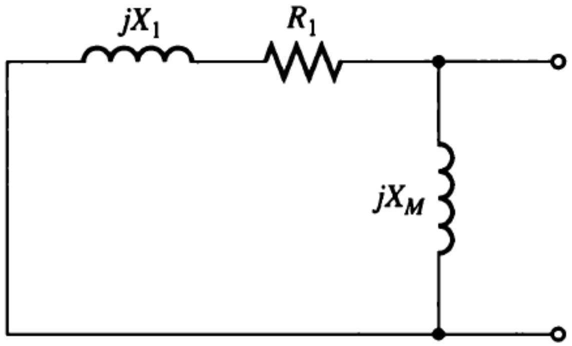
$$X_{TH} \approx X_1 \left(\frac{X_M}{X_1 + X_M}\right)$$

Hot to get?

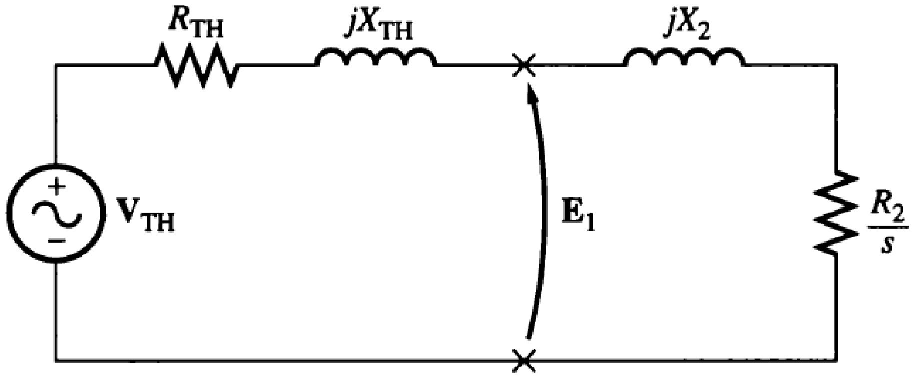
Induction Motor Torque-Speed Characteristics

$X_M >> X_1$ and $X_M >> R_1$

Short-circuit



Thevenin's equivalent



$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)}$$

$$= \frac{jR_1X_M - X_1X_M}{R_1 + j\left(X_1 + X_M\right)} \times \frac{R_1 - j\left(X_1 + X_M\right)}{R_1 - j\left(X_1 + X_M\right)}$$

$$= \frac{jR_1^2X_M + R_1X_M\left(X_1 + X_M\right) - R_1X_1X_M + jX_1X_M\left(X_1 + X_M\right)}{R_1^2 + \left(X_1 + X_M\right)^2}$$

$$= \frac{R_1X_M\left(X_1 + X_M\right) - R_1X_1X_M}{R_1^2 + \left(X_1 + X_M\right)^2} + j\frac{R_1^2X_M + X_1X_M\left(X_1 + X_M\right)}{R_1^2 + \left(X_1 + X_M\right)^2}$$

$$= \frac{R_1X_1X_M + R_1X_M^2 - R_1X_1X_M}{R_1^2 + \left(X_1 + X_M\right)^2} + j\frac{R_1^2X_M + X_1^2X_M + X_1X_M^2}{R_1^2 + \left(X_1 + X_M\right)^2}$$

$$= \frac{R_1X_M^2}{R_1^2 + \left(X_1 + X_M\right)^2} + j\frac{R_1^2X_M + X_1^2X_M + X_1X_M^2}{R_1^2 + \left(X_1 + X_M\right)^2}$$

If $X_M >> \mathrm{R}_1$

$$Z_{TH} = \frac{R_1X_M^2}{R_1^2 + \left(X_1 + X_M\right)^2} + j\frac{R_1^2X_M + X_1^2X_M + X_1X_M^2}{R_1^2 + \left(X_1 + X_M\right)^2}$$

$$\approx \frac{R_1X_M^2}{\left(X_1 + X_M\right)^2} + j\frac{X_1X_M\left(\frac{R_1^2}{X_1} + X_1 + X_M\right)}{\left(X_1 + X_M\right)^2}$$

$$\approx R_1\frac{X_M^2}{\left(X_1 + X_M\right)^2} + j\frac{X_1X_M\left(X_1 + X_M\right)}{\left(X_1 + X_M\right)^2}$$

$$\approx R_1\frac{X_M^2}{\left(X_1 + X_M\right)^2} + jX_1\frac{X_M}{\left(X_1 + X_M\right)}$$

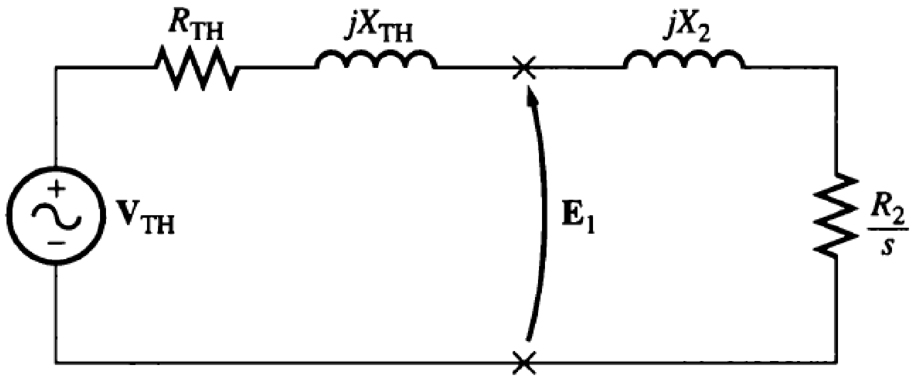
Induction Motor Torque-Speed Characteristics

- The magnitude of I_2 can be obtained

$$I_2 = \frac{V_{TH}}{R_{TH} + R_2/s + jX_{TH} + jX_2}$$

$$I_2 = \frac{V_{TH}}{\sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}}$$

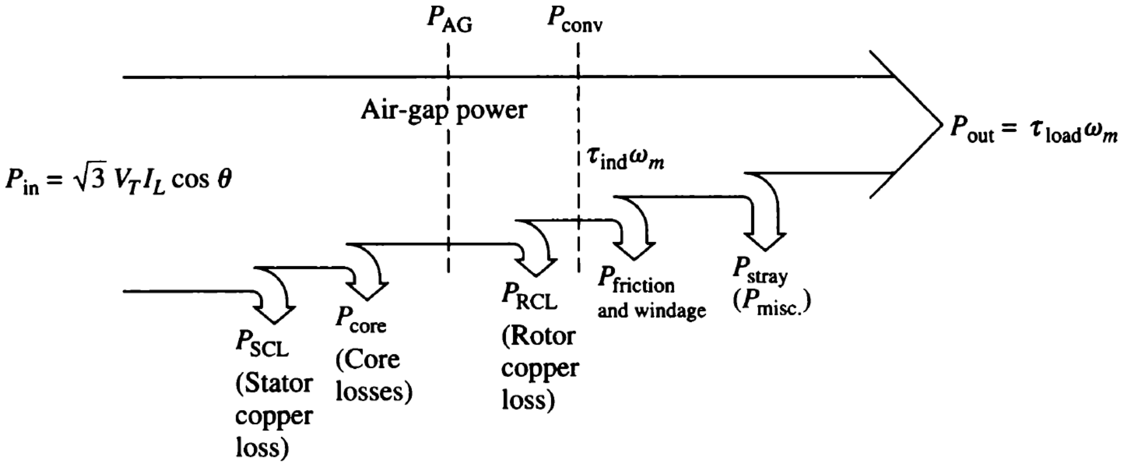
Thevenin's equivalent



- The air-gap power is

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

$$= \frac{3V_{TH}^2 R_2/s}{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$$



Induction Motor Torque-Speed Characteristics

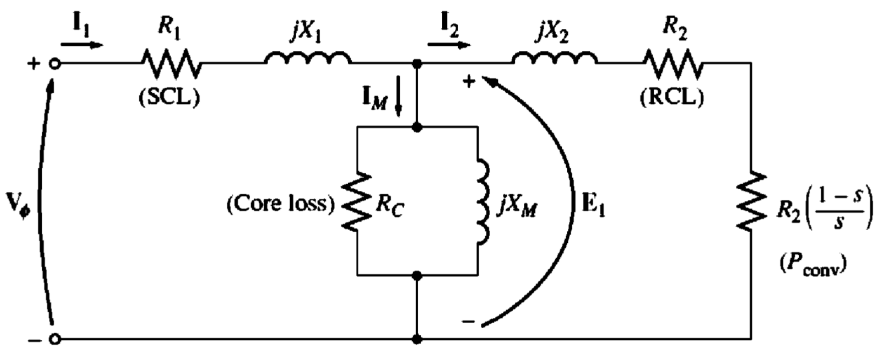
- Refer to the equivalent circuit, it is possible to derive a general expression for induced torque as a function of speed.

- The induced torque in an induction motor is given by

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{\omega_{sync} - s\omega_{sync}} = \frac{3I_2^2 R_2}{s\omega_{sync}} = \frac{P_{AG}}{\omega_{sync}}$$

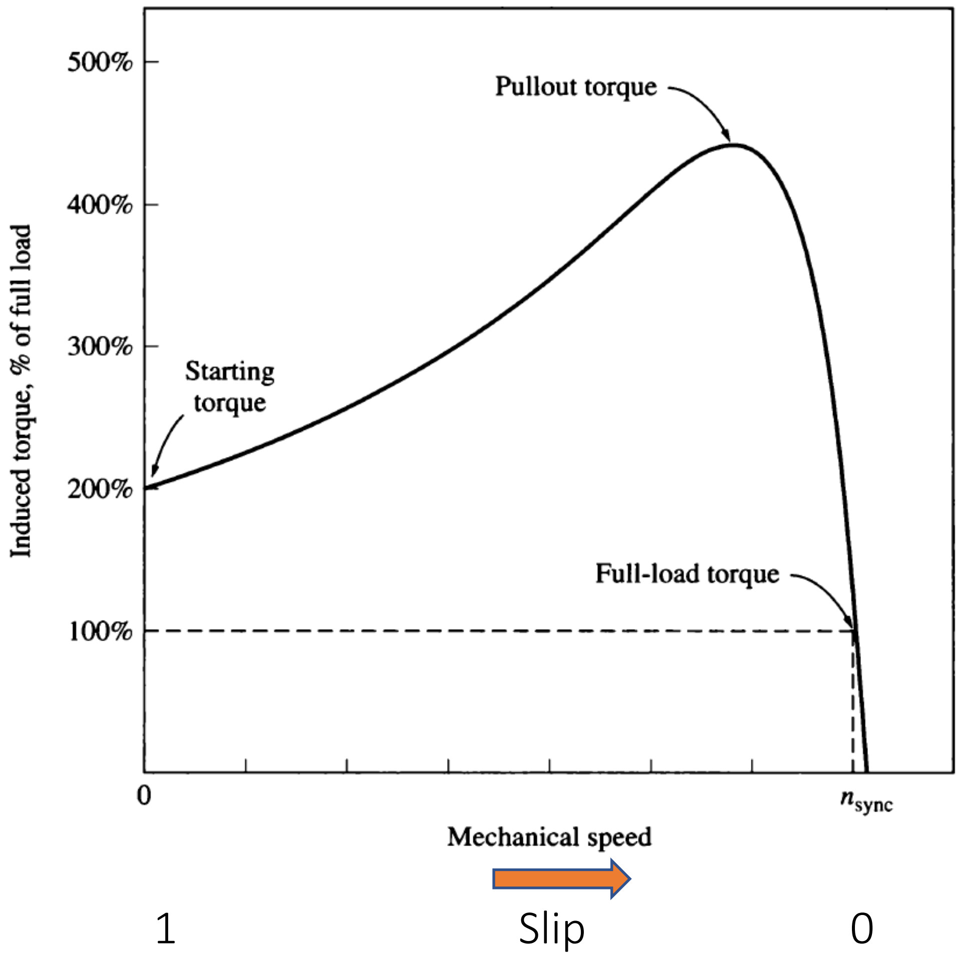
$$P_{conv} = 3I_2^2 R_2\left(\frac{1-s}{s}\right)$$

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = P_{RCL} + P_{conv}$$



Induction Motor Torque-Speed Characteristics

- The induced torque of the motor is zero at synchronous speed
- The torque-speed curve is nearly linear between no load and full load. In this range, the rotor resistance is much larger than the rotor reactance, so the rotor current, the rotor magnetic field, and the induced torque increase linearly with increasing slip.
- There is a maximum possible torque that cannot be exceeded, pullout torque or breakdown torque, is 2 to 3 times the rated full-load torque of the motor.
- The starting torque on the motor is slightly larger than its full-load torque, so this motor will start carrying any load that it can supply at full power.



Maximum (Pullout) Torque in an Induction Motor

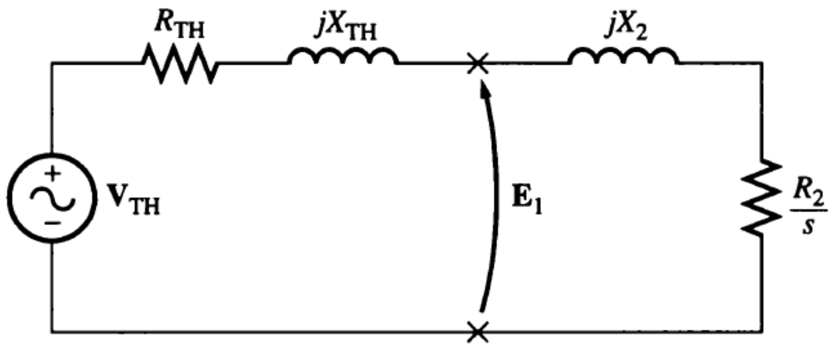
- When is the power supplied to R_2/s at its maximum?

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

$$= \frac{3V_{TH}^2 R_2 / s}{\left(R_{TH} + R_2 / s\right)^2 + \left(X_{TH} + X_2\right)^2}$$

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}}$$

$$\tau_{ind} = \frac{3V_{TH}^2 R_2/s}{\omega_{sync}[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2]}$$

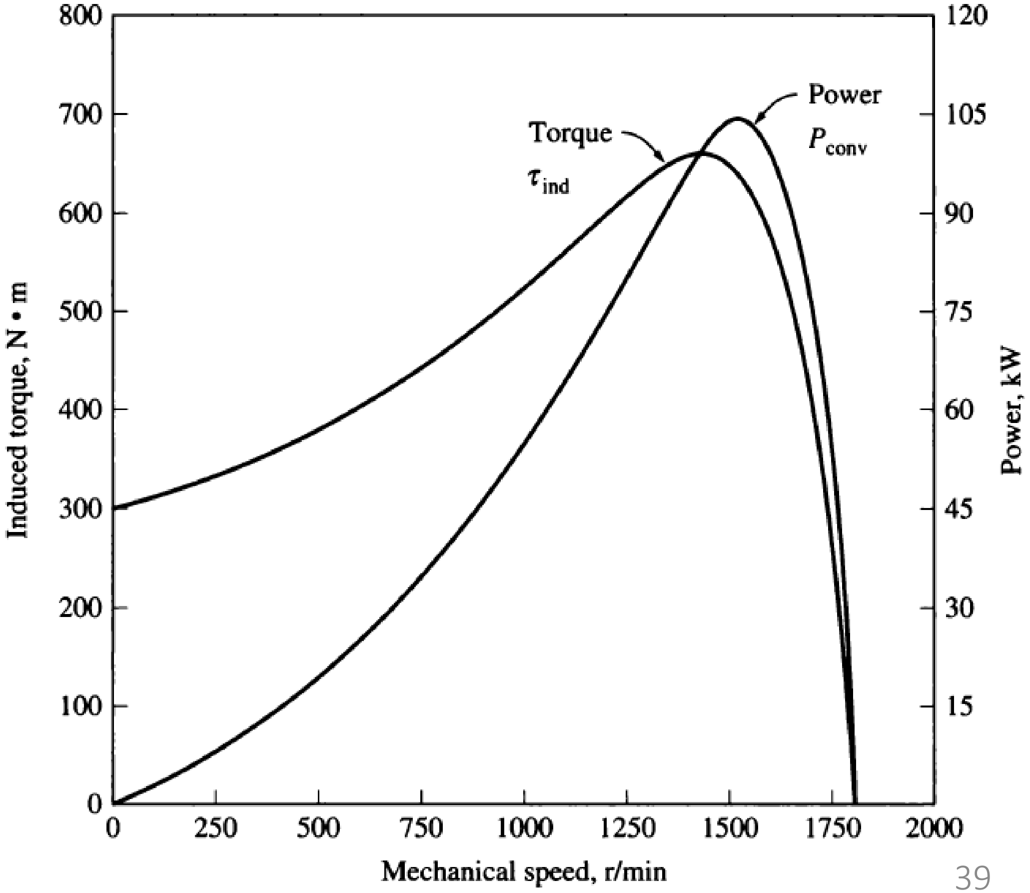


- Maximum power transfer to the load resistor R_2/s will occur when the magnitude of that impedance is equal to the magnitude of the source impedance.

$$\frac{R_2}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}$$

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}$$



Example 4: Torque Calculation for Induction Motor

- A 460-V, 25-hp, 60-Hz, four-pole, Y-connected wound-rotor induction motor has the following impedances in ohms per phase referred to the stator circuit:

  R_1=0.641, R_2=0.332, X_1=1.106, X_2=0.464, X_M=26.3

  The total rotational losses are 1100 W and are assumed to be constant. The core loss is lumped in with the rotational losses. For a rotor slip of 2.2 percent at the rated voltage and rated frequency, find the motor's

  a) What is the maximum torque of this motor? At what speed and slip does it occur?
  b) What is the starting torque of this motor?
  c) When the rotor resistance is doubled, what is the speed at which the maximum torque now occurs? What is the new starting torque of the motor?

Example 4: Torque Calculation for Induction Motor

❑ Solution

The Thevenin voltage of this motor is

$$V_{TH} = V_\phi \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$= \frac{(266\ \text{V})(26.3\ \Omega)}{\sqrt{(0.641\ \Omega)^2 + (1.106\ \Omega + 26.3\ \Omega)^2}} = 255.2\ \text{V}$$

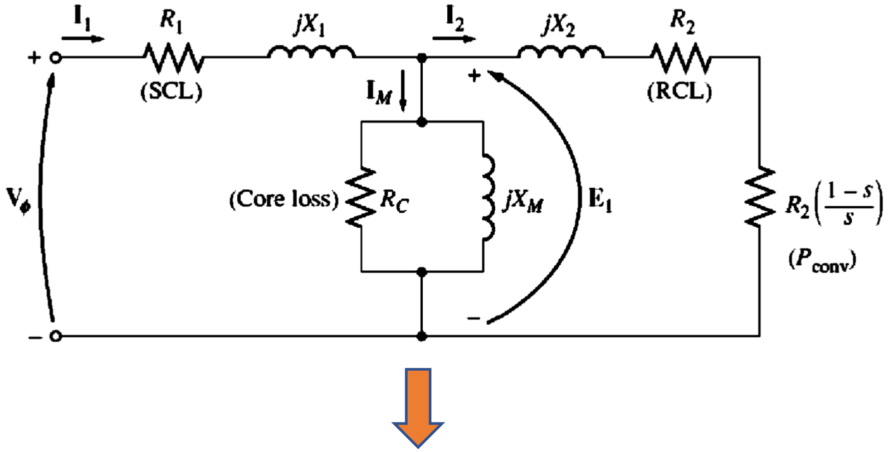
The Thevenin resistance is

$$R_{TH} \approx R_1 \left(\frac{X_M}{X_1 + X_M}\right)^2$$

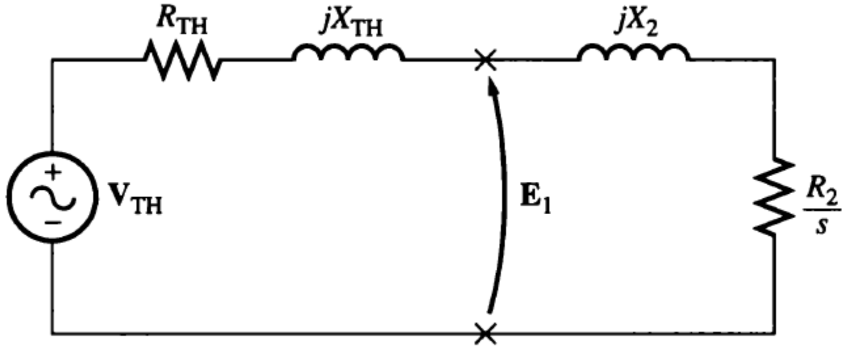
$$\approx (0.641\ \Omega)\left(\frac{26.3\ \Omega}{1.106\ \Omega + 26.3\ \Omega}\right)^2 = 0.590\ \Omega$$

The Thevenin reactance is

$$X_{TH} \approx X_1 = 1.106\ \Omega$$



Thevenin's equivalent



Resistor R_C represents core losses, but will not join any calculations.

Example 4: Torque Calculation for Induction Motor

❑ Solution

(a) The slip at which maximum torque occurs is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$= \frac{0.332\ \Omega}{\sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}} = 0.198$$

This corresponds to a mechanical speed of

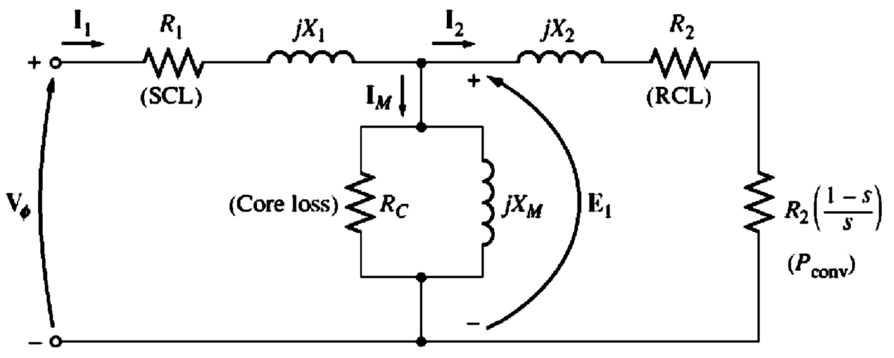
$$n_m = (1 - s)n_{sync} = (1 - 0.198)(1800\ \text{r/min}) = 1444\ \text{r/min}$$

The torque at this speed is

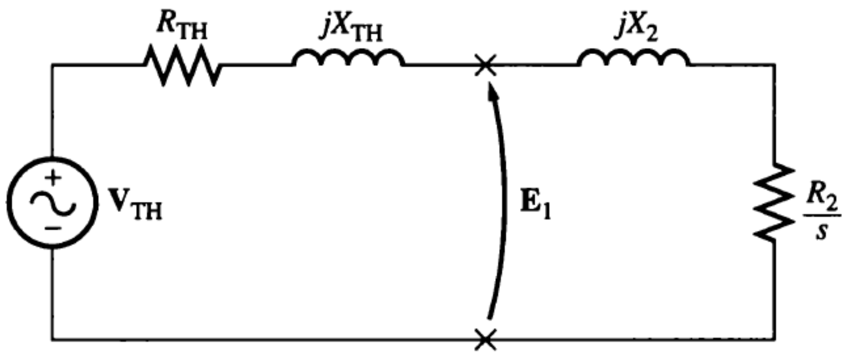
$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}]}$$

$$= \frac{3(255.2\ \text{V})^2}{2(188.5\ \text{rad/s})[0.590\ \Omega + \sqrt{(0.590\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2}]}$$

$$= 229\ \text{N}\cdot\text{m}$$



Thevenin's equivalent



Example 4: Torque Calculation for Induction Motor

❑ Solution

(b) The starting torque of this motor is found by setting s = 1

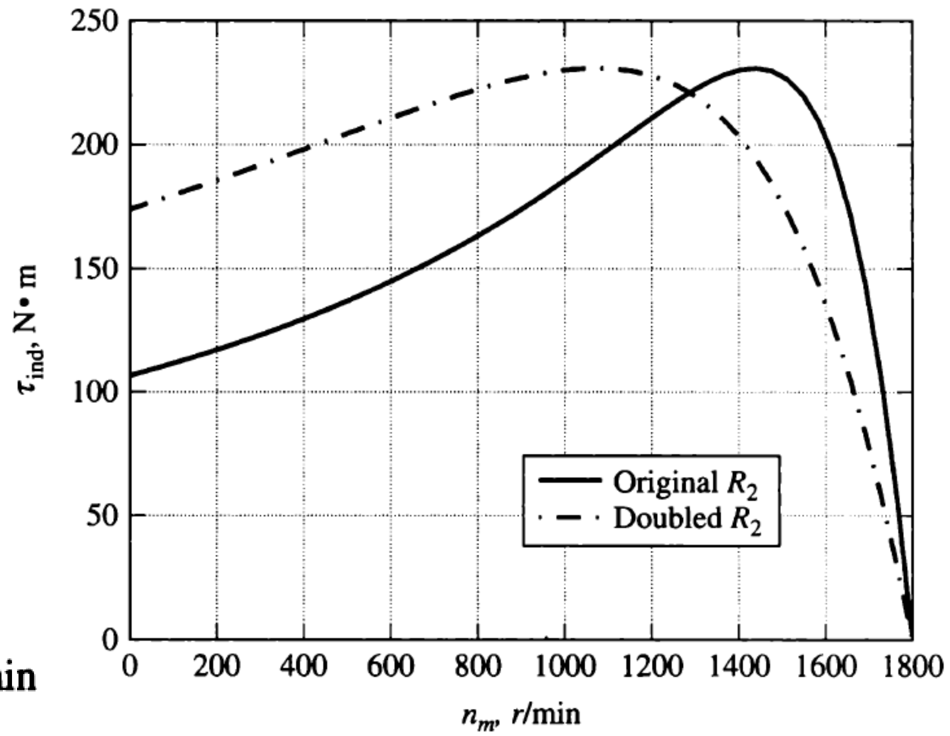
$$\tau_{start} = \frac{3V_{TH}^2 R_2}{\omega_{sync}[(R_{TH} + R_2)^2 + (X_{TH} + X_2)^2]}$$

$$= \frac{3(255.2\text{ V})^2(0.332\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.332\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$$

$$= 104\text{ N}\bullet\text{m}$$

(c) If the rotor resistance is doubled, then the slip at maximum torque doubles, too. Therefore,

$$s_{max} = 0.396 \qquad n_m = (1 - s)n_{sync} = (1 - 0.396)(1800\text{ r/min}) = 1087\text{ r/min}$$



The maximum torque is still $\tau_{max} = 229\text{ N}\bullet\text{m}$

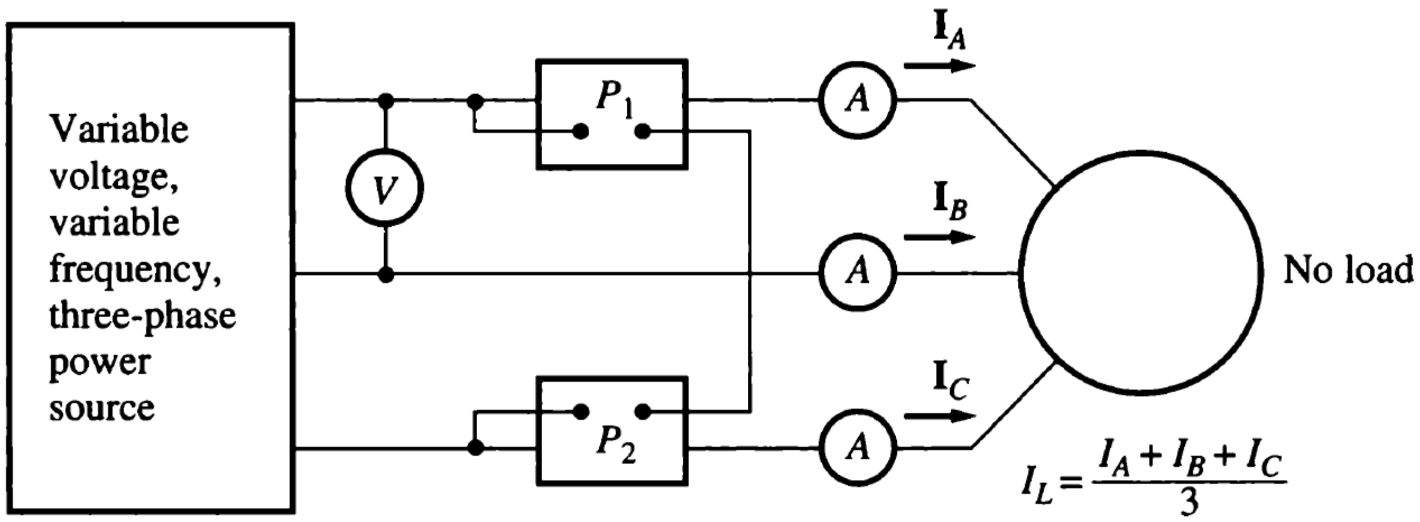
The starting torque is now $\tau_{start} = \frac{3(255.2\text{ V})^2(0.664\ \Omega)}{(188.5\text{ rad/s})[(0.590\ \Omega + 0.664\ \Omega)^2 + (1.106\ \Omega + 0.464\ \Omega)^2]}$

$$= 170\text{ N}\bullet\text{m}$$

Determining Circuit Model Parameters: No-load Test

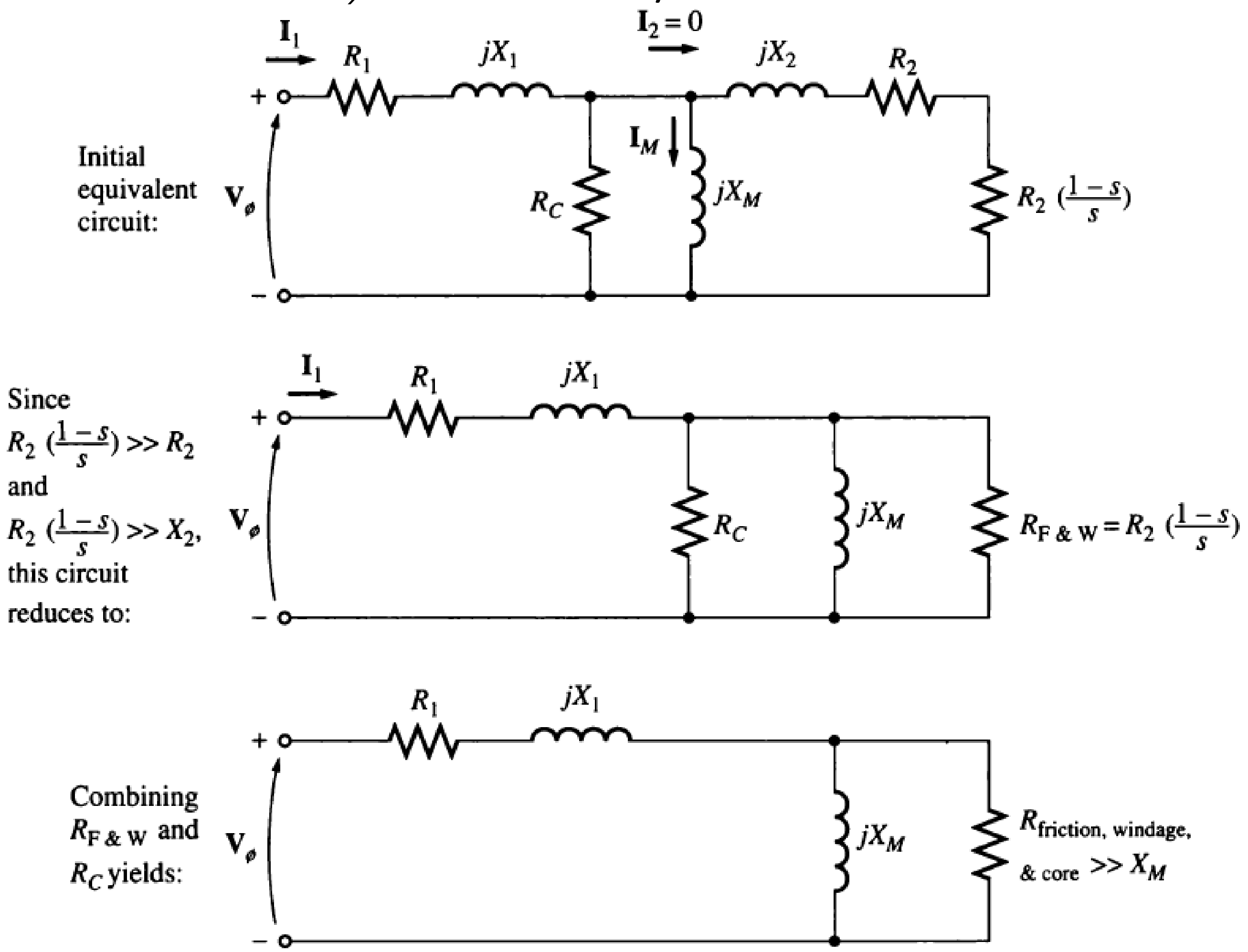
- Wattmeter, a voltmeter and three ammeters are connected to an induction motor.

- The only load on the motor is the friction and winding losses. Measures the rotational losses of the motor and provides information about its magnetization current.



No-Load Test

- In this motor at no-load conditions, the input power measured by the meters must equal the losses in the motor. The rotor copper losses are negligible because the current I_2 is extremely small.



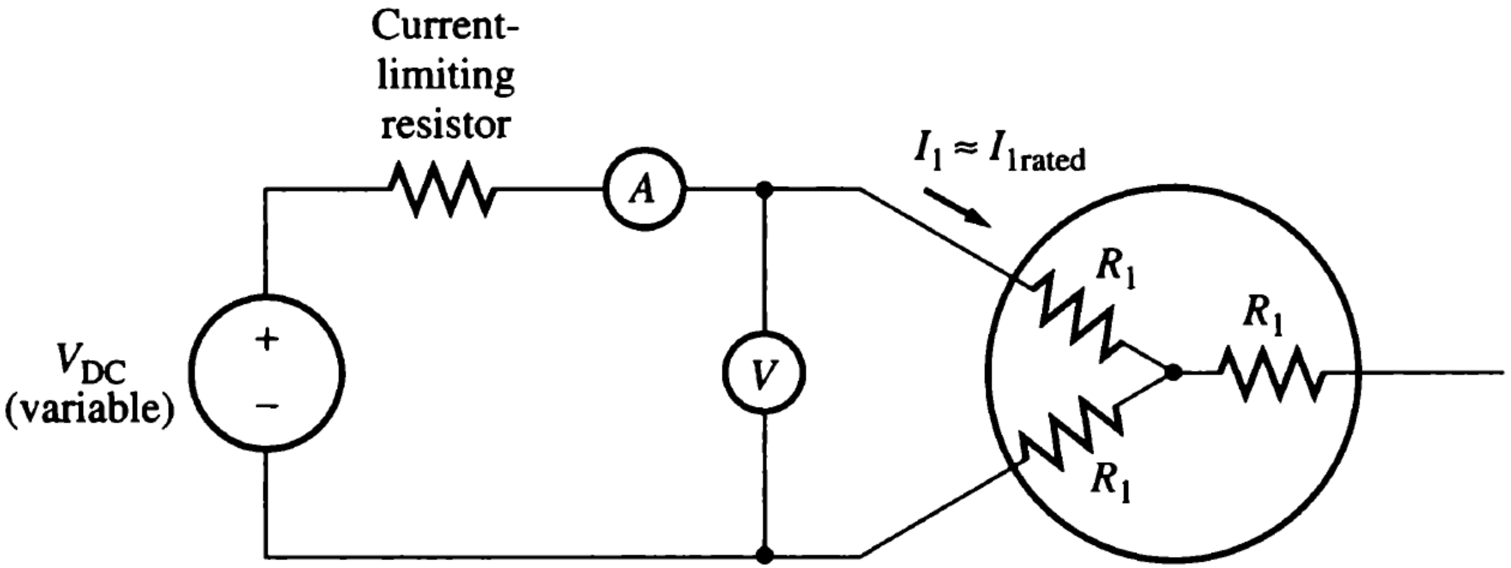
$$P_{NLT} = P_{SCL} + P_{core} + P_{F\&W}$$

$$|Z_{NLT}| = V_{\phi}/I_1$$

$$|Z_{eq}| = \frac{V_{\phi}}{I_{1,nl}} \approx X_1 + X_M$$

DC Test for Stator Resistance

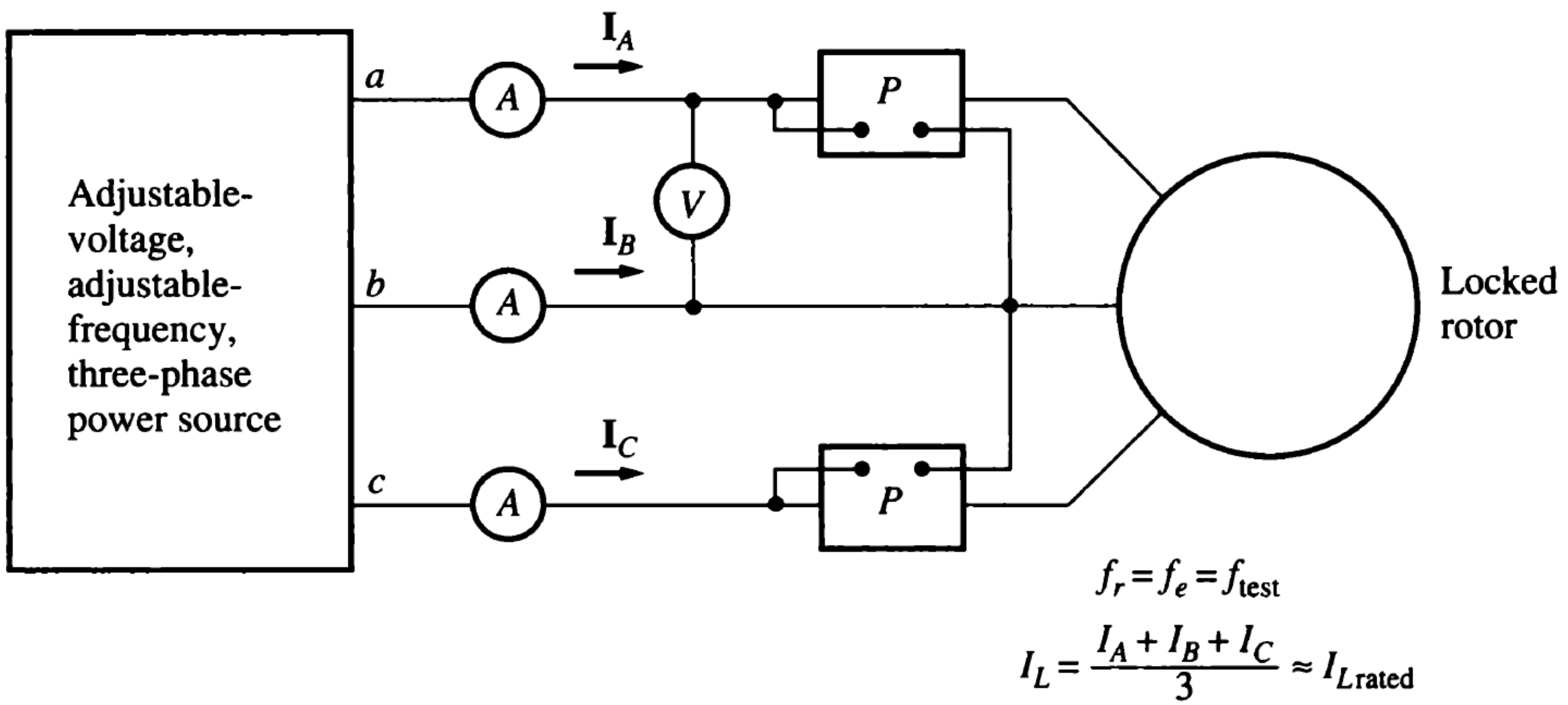
- A voltmeter and an ammeter are connected to an induction motor.
- A test for R_1 independent of R_2, X_1 and X_2.
- The current in the stator windings is adjusted to the rated value in an attempt to heat the windings to the same temperature they would have during normal operation.



$$2R_1 = \frac{V_{dc}}{I_{dc}}$$

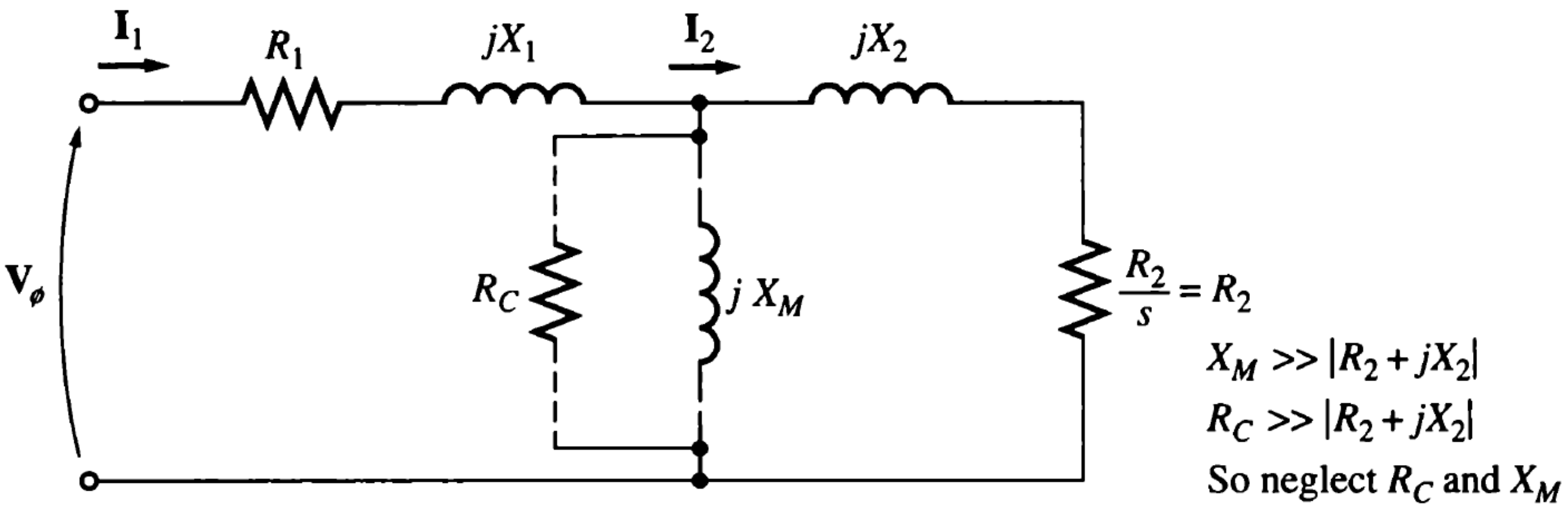
Locked-Rotor Test

- Similar to no-load test, wattmeter, a voltmeter and three ammeters are connected to an induction motor.
- This test corresponds to the short-circuit test on a transformer. Therefore, parameters are measured before the rotor can heat up too much.



Locked-Rotor Test

- Since the rotor is not moving, s=1.
- A problem in that the line frequency does not represent the normal operating conditions of the rotor.
- A typical compromise is to use a frequency 25 percent or less of the rated frequency.



Determining Circuit Model Parameters

- No-Load Test

$$P_{NLT} = P_{SCL} + P_{core} + P_{F\&W}$$
$$= 3I_1^2 R_1 + P_{core+F\&W}$$

$$\cos\theta_{NLT} = \frac{P_{NLT}}{\sqrt{3}V_{LL}I_L}$$

$$Z_{NLT} = R_1 + jX_1 + jX_m \parallel R_{core+F\&W}$$
$$\approx R_1 + j(X_1 + X_m)$$
$$\approx |Z_{NLT}|cos\theta_{NLT} + j|Z_{NLT}|sin\theta_{NLT}$$

- DC Test for Stator Resistance

$$2R_1 = \frac{V_{dc}}{I_{dc}} \qquad R_1 = \frac{V_{dc}}{2I_{dc}}$$

- Locked-Rotor Test

$$P_{LRT} = P_{SCL} + P_{RCL}$$

$$\cos\theta_{LRT} = \frac{P_{LRT}}{\sqrt{3}V_{LL}I_L}$$

$$Z_{LRT} = (R_1 + R_2) + j(X_1 + X_2)$$
$$= |Z_{LRT}|\cos\theta_{LRT} + j|Z_{LRT}|\sin\theta_{LRT}$$

Example: Determining Circuit Model Parameters

The following test data were taken on a 7.5-hp, four-pole, 208-V 60-Hz, design A, Y-connected induction motor having a rated current of 28 A.

DC test:

$V_{dc} = 13.6$ V
$I_{dc} = 28.0$ A

No-load test:

$V_T = 208$ V
$f = 60$ Hz
$I_A = 8.12$A, $I_B = 8.20$ A, $I_C = 8.18$ A
Pin = 420 W

Locked-rotor test:

$V_T = 25$ V
$I_A = 28.1$ A, $I_B = 28.0$ A, $I_C = 27.6$ A
$f = 15$ Hz
Pin = 920 W

(a) Sketch the per-phase equivalent circuit for this motor.
(b) Find the slip at the pullout torque, and find the value of the pullout torque itself.

Example: Determining Circuit Model Parameters

Solution:

(a) From the dc test,

$$R_1 = \frac{V_{DC}}{2I_{DC}} = \frac{13.6\text{ V}}{2(28.0\text{ A})} = 0.243\ \Omega$$

From the no-load test,

$$I_{L,av} = \frac{8.12\text{ A} + 8.20\text{ A} + 8.18\text{ A}}{3} = 8.17\text{ A}$$

$$V_{\phi,nl} = \frac{208\text{ V}}{\sqrt{3}} = 120\text{ V}$$

Therefore,

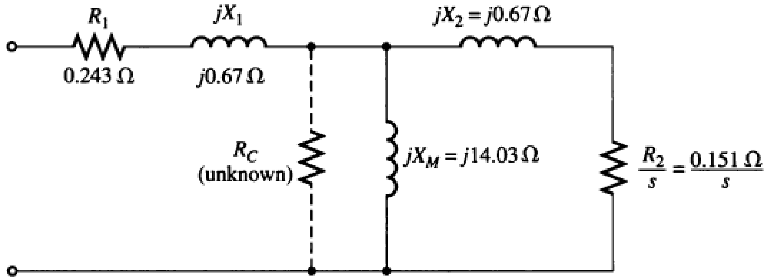
$$|Z_{nl}| = \frac{120\text{ V}}{8.17\text{ A}} = 14.7\ \Omega = X_1 + X_M$$

When X_1 is known, X_M can be found. The stator copper losses are

$$P_{SCL} = 3I_1^2 R_1 = 3(8.17\text{ A})^2(0.243\ \Omega) = 48.7\text{ W}$$

Therefore, the no-load rotational losses are

$$P_{rot} = P_{in,nl} - P_{SCL,nl}$$
$$= 420\text{ W} - 48.7\text{ W} = 371.3\text{ W}$$



From the locked-rotor test,

$$I_{L,av} = \frac{28.1\text{ A} + 28.0\text{ A} + 27.6\text{ A}}{3} = 27.9\text{ A}$$

The locked-rotor impedance is

$$|Z_{LR}| = \frac{V_\phi}{I_A} = \frac{V_T}{\sqrt{3}I_A} = \frac{25\text{ V}}{\sqrt{3}(27.9\text{ A})} = 0.517\ \Omega$$

and the impedance angle θ is

$$\theta = \cos^{-1}\frac{P_{in}}{\sqrt{3}V_T I_L}$$
$$= \cos^{-1}\frac{920\text{ W}}{\sqrt{3}(25\text{ V})(27.9\text{ A})}$$
$$= \cos^{-1} 0.762 = 40.4°$$

Therefore, $R_{LR} = 0.517 \cos 40.4° = 0.394\ \Omega = R_1 + R_2$. Since $R_1 = 0.243\ \Omega$, R_2 must be 0.151 Ω. The reactance at 15 Hz is

$$X'_{LR} = 0.517 \sin 40.4° = 0.335\ \Omega$$

The equivalent reactance at 60 Hz is

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) 0.335\ \Omega = 1.34\ \Omega$$

For design class A induction motors, this reactance is assumed to be divided equally between the rotor and stator, so

$$X_1 = X_2 = 0.67\ \Omega$$
$$X_M = |Z_{nl}| - X_1 = 14.7\ \Omega - 0.67\ \Omega = 14.03\ \Omega$$

Example: Determining Circuit Model Parameters

Solution:

(b) For this equivalent circuit, the Thevenin equivalents are found from Equations

$$V_{TH} = 114.6 \text{ V} \qquad R_{TH} = 0.221\ \Omega \qquad X_{TH} = 0.67\ \Omega$$

Therefore, the slip at the pullout torque is given by

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

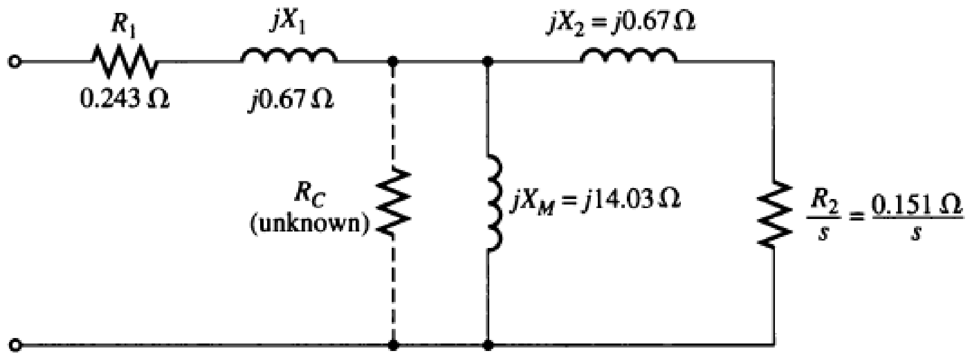
$$= \frac{0.151\ \Omega}{\sqrt{(0.243\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}} = 0.111 = 11.1\%$$

The maximum torque of this motor is given by

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X^2)}]}$$

$$= \frac{3(114.6\text{ V})^2}{2(188.5\text{ rad/s})[0.221\ \Omega + \sqrt{(0.221\ \Omega)^2 + (0.67\ \Omega + 0.67\ \Omega)^2}]}$$

$$= 66.2 \text{ N} \bullet \text{m}$$



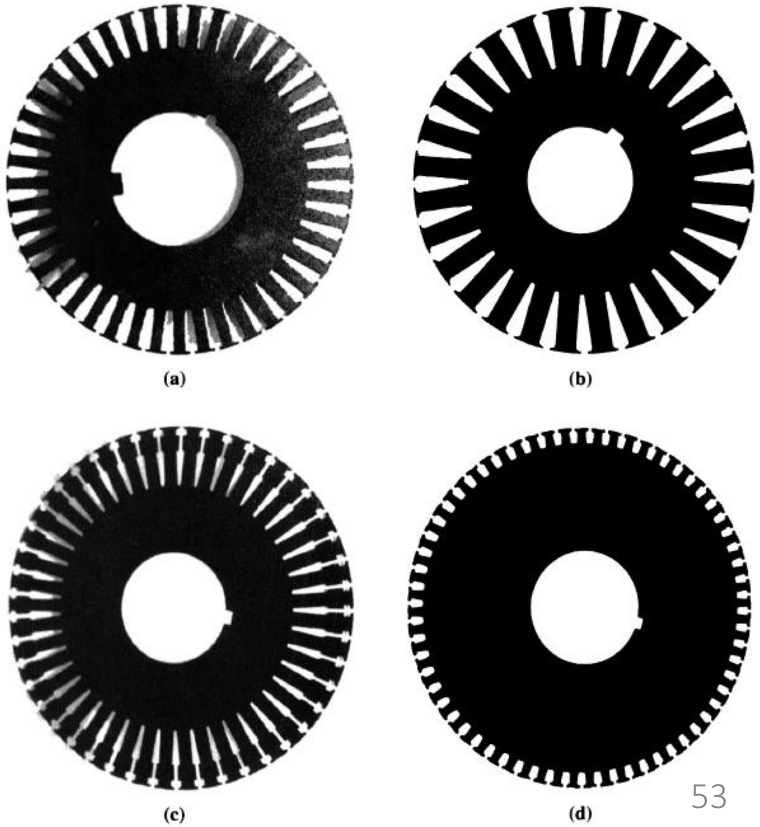
Determining Circuit Model Parameters

- No simple way to separate the contributions of stator and rotor reactances.
- In normal practice, it does not matter just how X_1 and X_2 is broken down. Since the reactance appears as the sum $X_1 + X_2$ in all the torque equations.
- Experience has shown that motors of certain design types have certain proportions between the rotor and stator reactances.

	X_1 and X_2 as functions of X_{LR}	
Rotor Design	X_1	X_2
Wound rotor	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design A	$0.5\ X_{LR}$	$0.5\ X_{LR}$
Design B	$0.4\ X_{LR}$	$0.6\ X_{LR}$
Design C	$0.3\ X_{LR}$	$0.7\ X_{LR}$
Design D	$0.5\ X_{LR}$	$0.5\ X_{LR}$

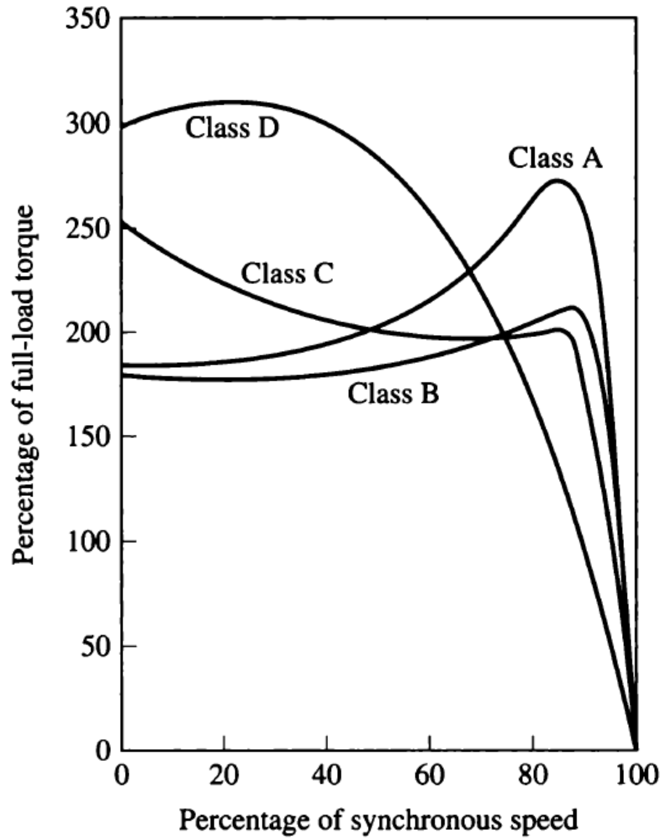
Rules of thumb for dividing rotor and stator circuit reactance.

$$X_{LR} = \frac{f_{\text{rated}}}{f_{\text{test}}} X'_{LR} = X_1 + X_2$$



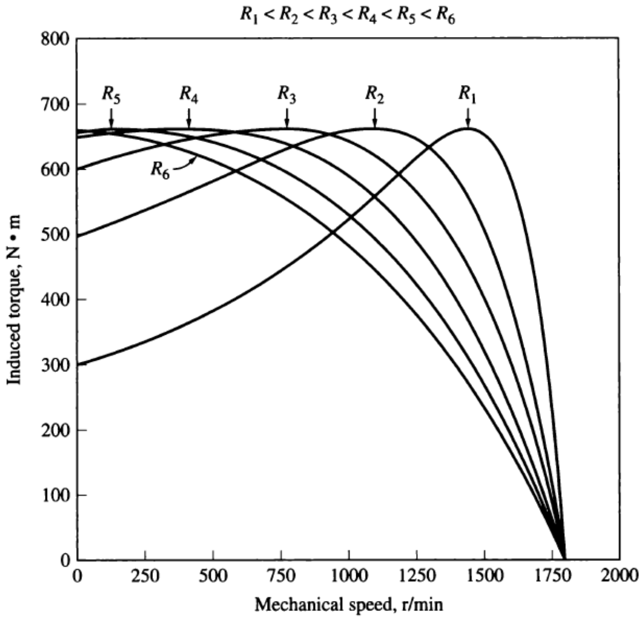
Class of Motors and Their Applications

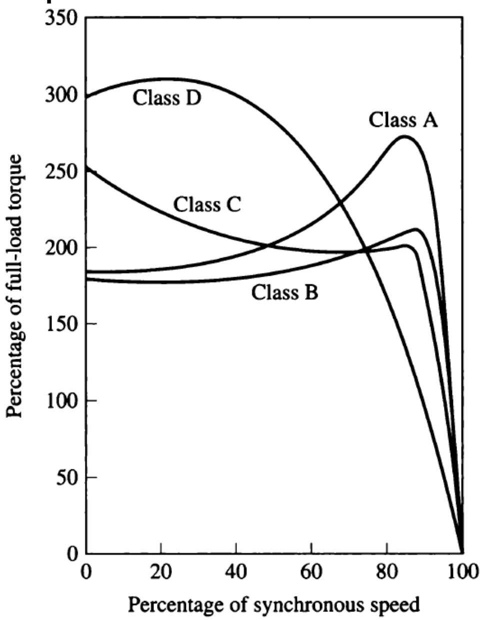
- **Class A** – Normal starting torque, normal starting current, normal slip. It is the most popular type of cage induction motor. They are used in fans, compressors, pumps, conveyors etc which are having low inertia loads so that the motor can accelerate in less time.
- **Class B** – Normal starting torque, low starting current, normal slip. They are used where load is having high inertia. e.g. large fans, machine tools applications, for driving electric generators, centrifugal pumps, etc.
- **Class C** – High starting torque, low starting current, normal slip. They are generally double cage type. These motors are used where sufficiently high starting torque with reduced starting voltage is required. Eg. Crushers, compression pumps, large refrigerators, extile machinery, wood working equipment etc.
- **Class D**- High starting torque, low starting current, high slip. Full load slip may vary from 5% to 20% depending upon application. eg. bulldozers, shearing machines, metal drawing equipment, laundry equipment etc.



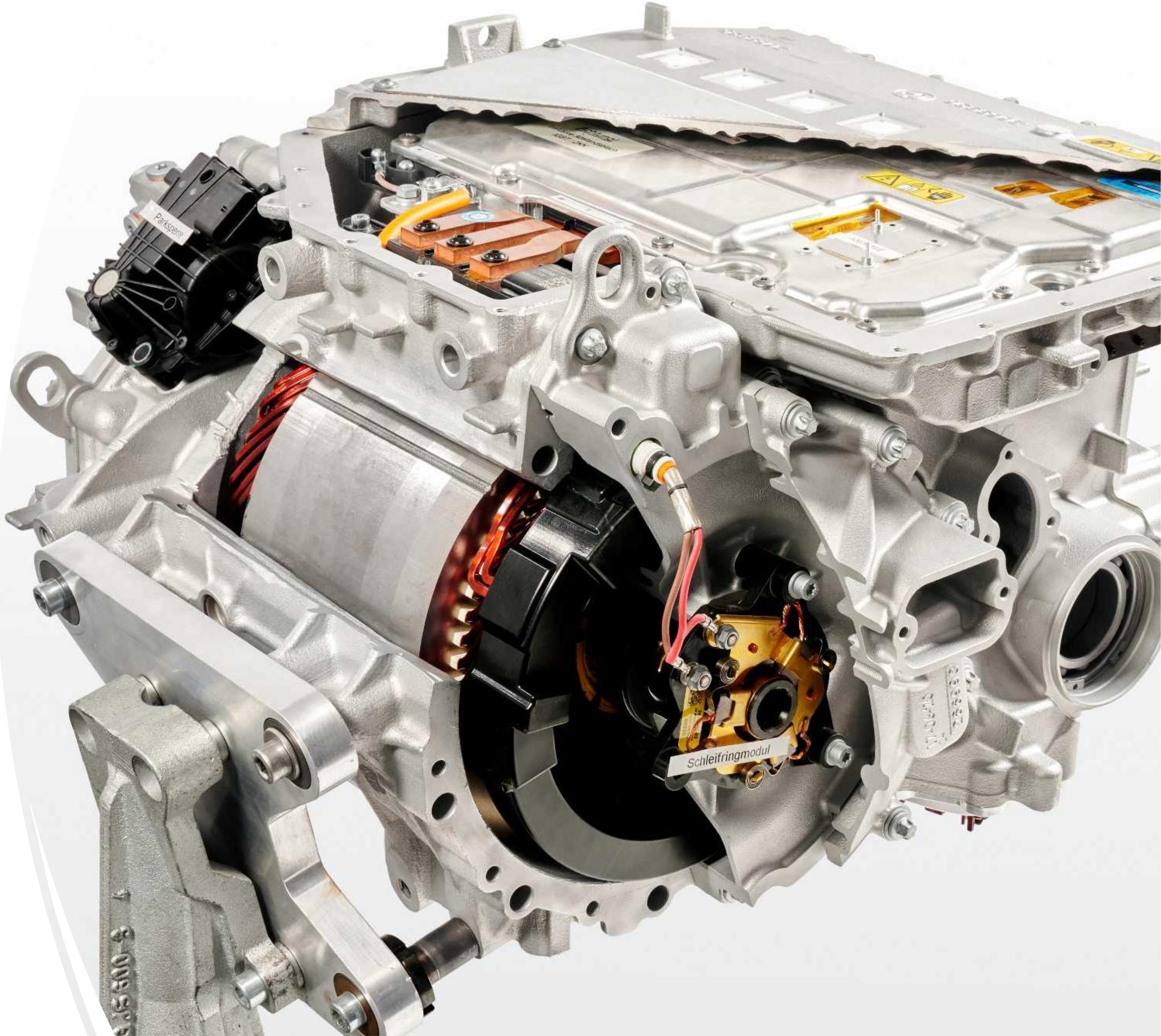
Summary

- Torque is proportional to the square of the supply voltage
- Torque is inversely related to the size of the stator impedances and rotor reactances.
- The smaller machine's reactances, the larger the maximum torque .
- Slip at which the maximum torque occurs is directly proportional to rotor resistance.
- The value of the maximum torque is independent of the value of rotor resistance.





Induction Motor Control

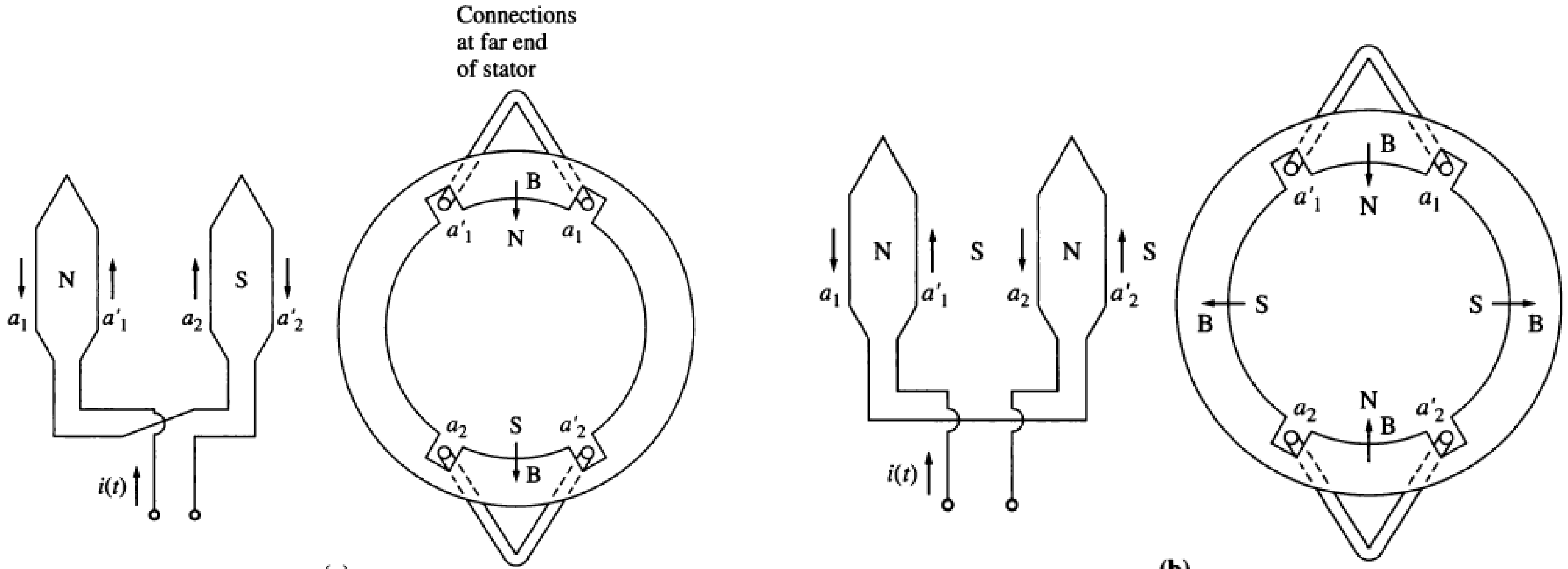


Speed Control by Pole Changing

- There are two major approaches to changing the number of poles :
  1. The method of consequent poles
  2. Multiple stator windings

$$n_{sync} = \frac{120 f_{se}}{P}$$

- Consequent poles is quite an old method for speed control, which relies on the fact that the number of poles in the stator windings of an induction motor can easily be changed by a factor of 2:1 (also is the ratio of speed change - main drawback).

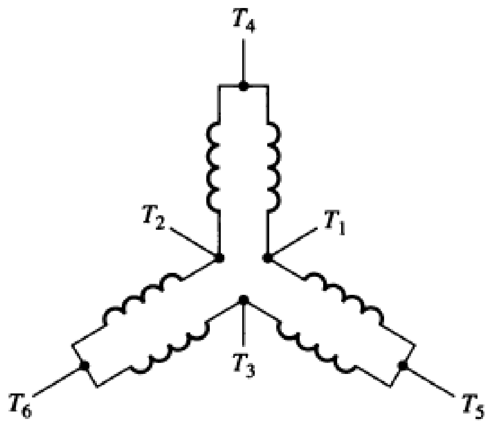


(a) (b)

Speed Control by Pole Changing

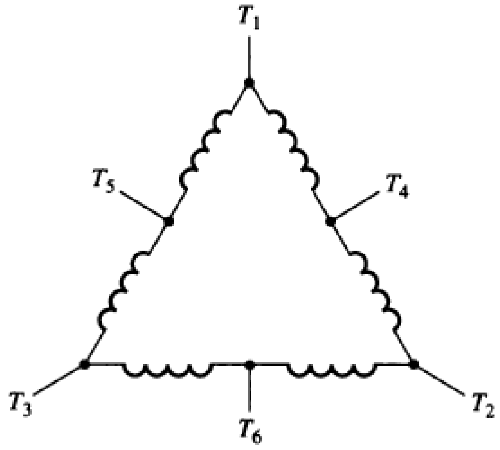
- By combining the method of consequent poles with multiple stator windings, it is possible to build a four-speed induction motor. For example, with separate four- and six-pole windings, it is possible to produce a 60-Hz motor capable of running at 600, 900, 1200, and 1800 r/min.

$$n_{sync} = \frac{120 f_{se}}{P}$$



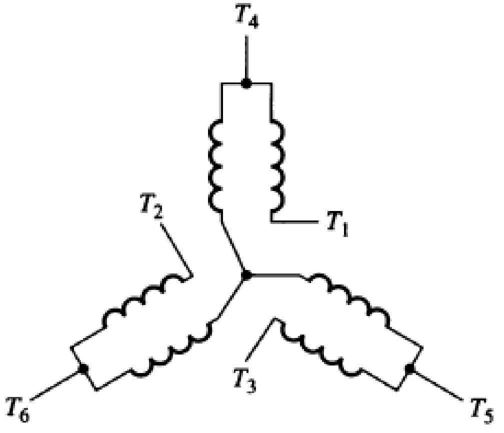
Speed	Lines			
	L_1	L_2	L_3	
Low	T_1	T_2	T_3	T_4, T_5, T_6 open
High	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together

(a)



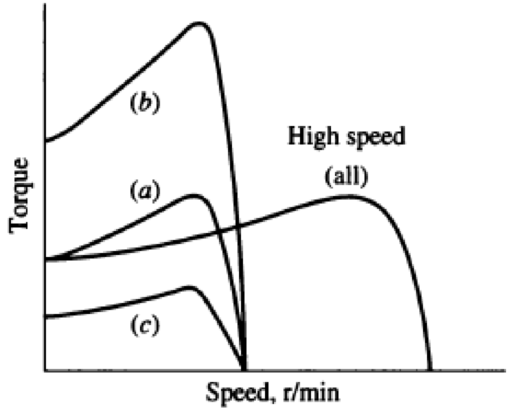
Speed	Lines			
	L_1	L_2	L_3	
Low	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together
High	T_1	T_2	T_3	T_4, T_5, T_6 open

(b)



Speed	Lines			
	L_1	L_2	L_3	
Low	T_1	T_2	T_3	T_4, T_5, T_6 open
High	T_4	T_5	T_6	$T_1 - T_2 - T_3$ together

(c)



(d)

Speed Control by Changing the Line Frequency

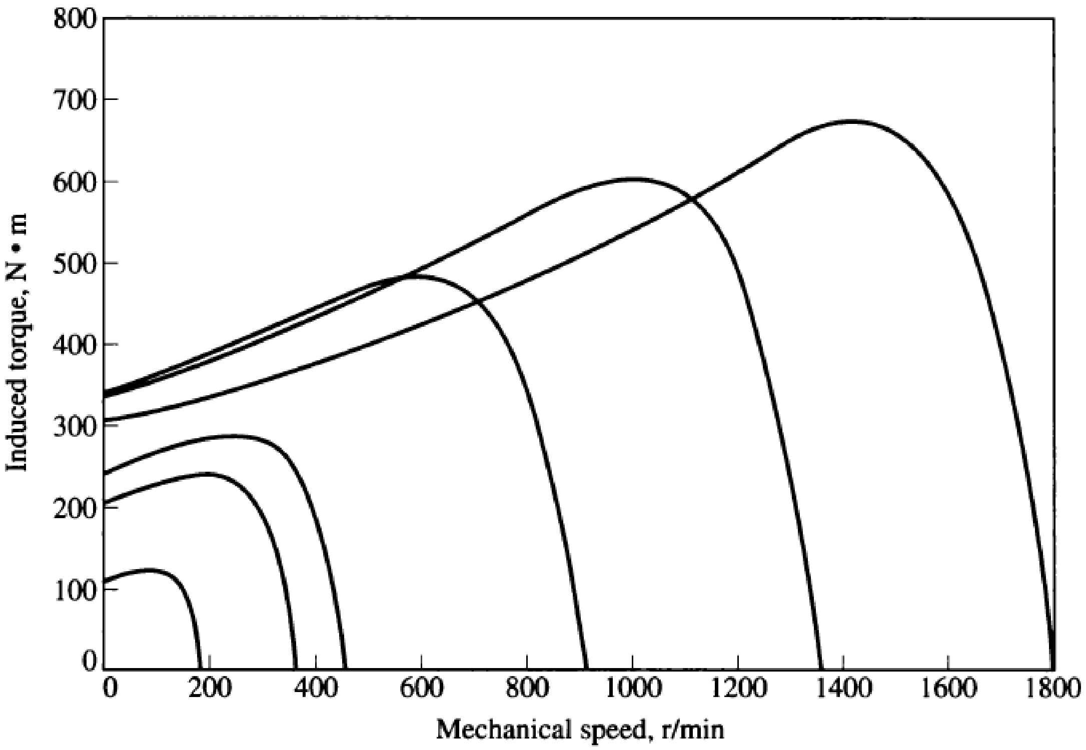
- If the electrical frequency applied to the stator of an induction motor is changed, the rate of rotation of its magnetic field n_{sync} will change in direct proportion to the change in electrical frequency, and the no-load point on the torque-speed characteristic curve will change with it.

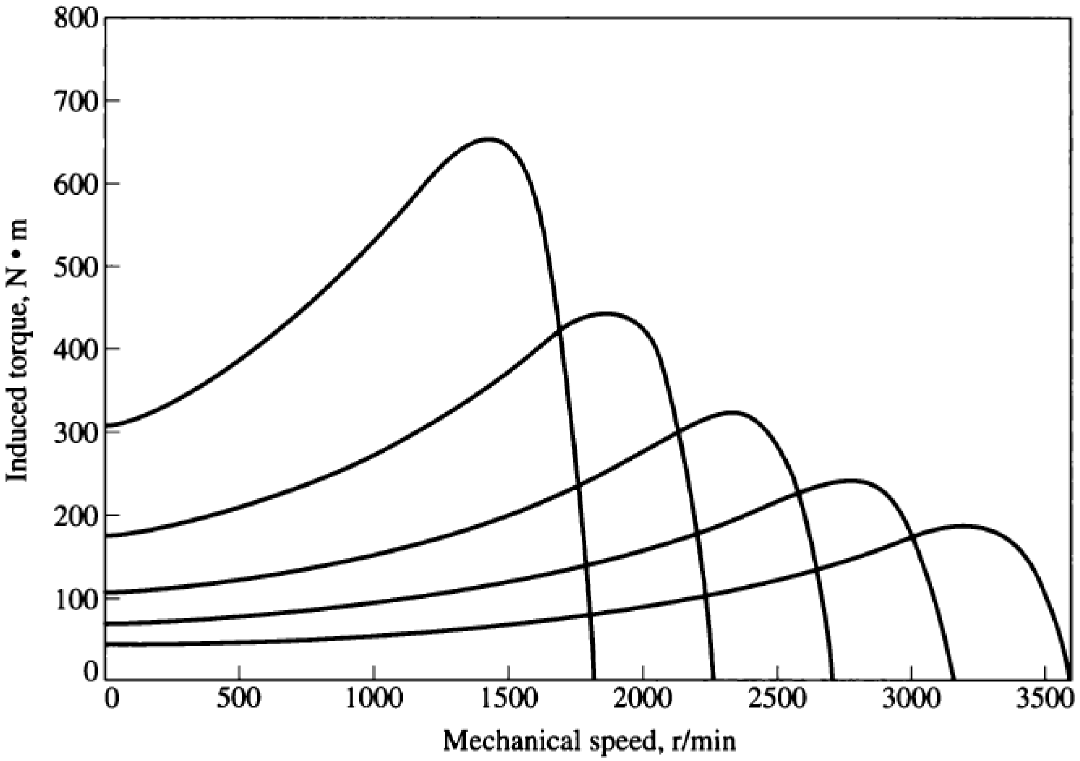
- The synchronous speed of the motor at rated conditions is known as the base speed. By using variable frequency control, it is possible to adjust the speed of the motor either above or below base speed.

- Variable-frequency induction motor drive is very useful. However, it is important to maintain certain voltage (avoid excessive magnetization current) and torque limits on the motor as the frequency is varied, to ensure safe operation.

Speed Control by Changing the Line Frequency

- (Left) The family of torque-speed characteristic curves for speeds below base speed, assuming that the line voltage is derated linearly with frequency.
- (Right) The family of torque-speed characteristic curves for speeds above base speed, assuming that the line voltage is held constant.

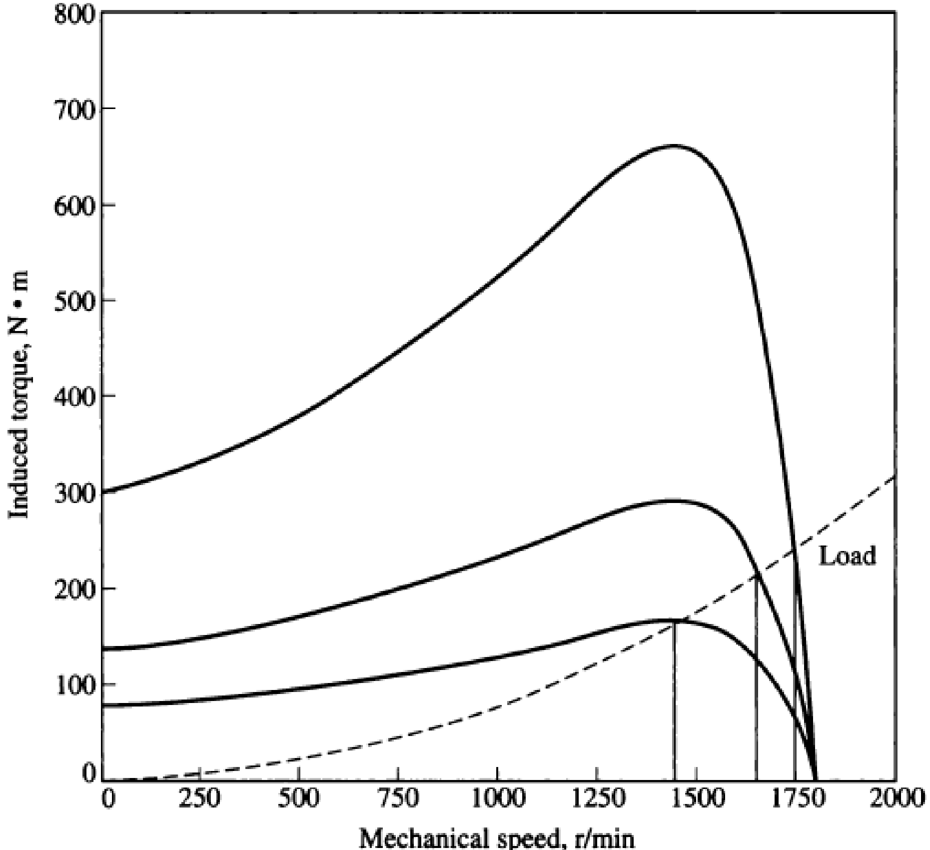




The synchronous speed of the motor at rated conditions is known as the *base speed.*

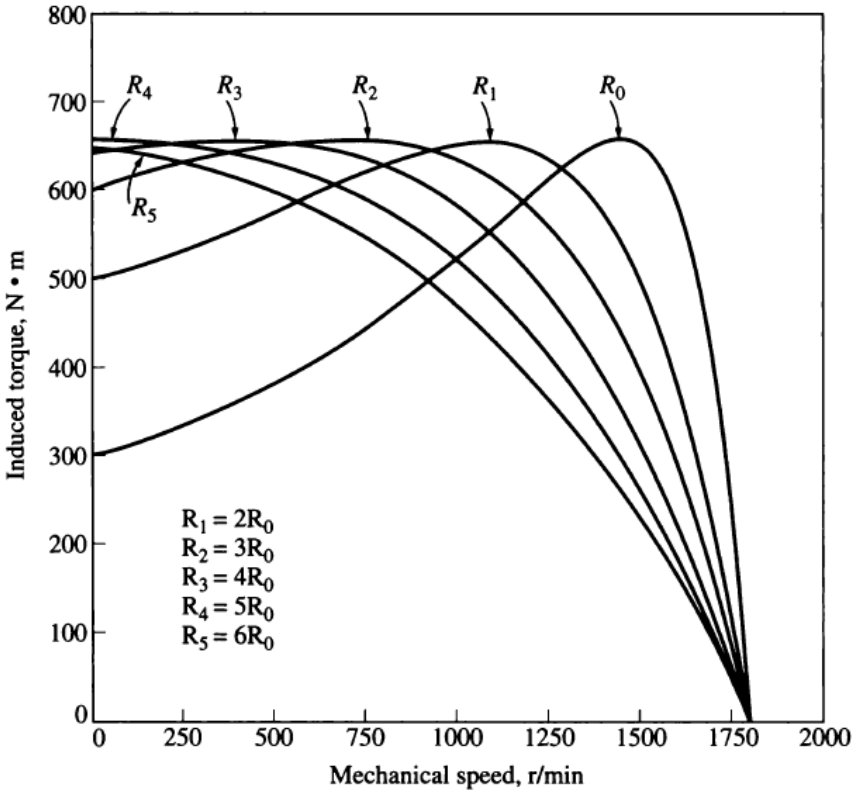
Speed Control by Changing the Line Voltage

- The torque developed by an induction motor is proportional to the square of the applied voltage.
- The speed of the motor may be controlled over a limited range by varying the line voltage. This method of speed control is sometimes used on small motors driving fans.



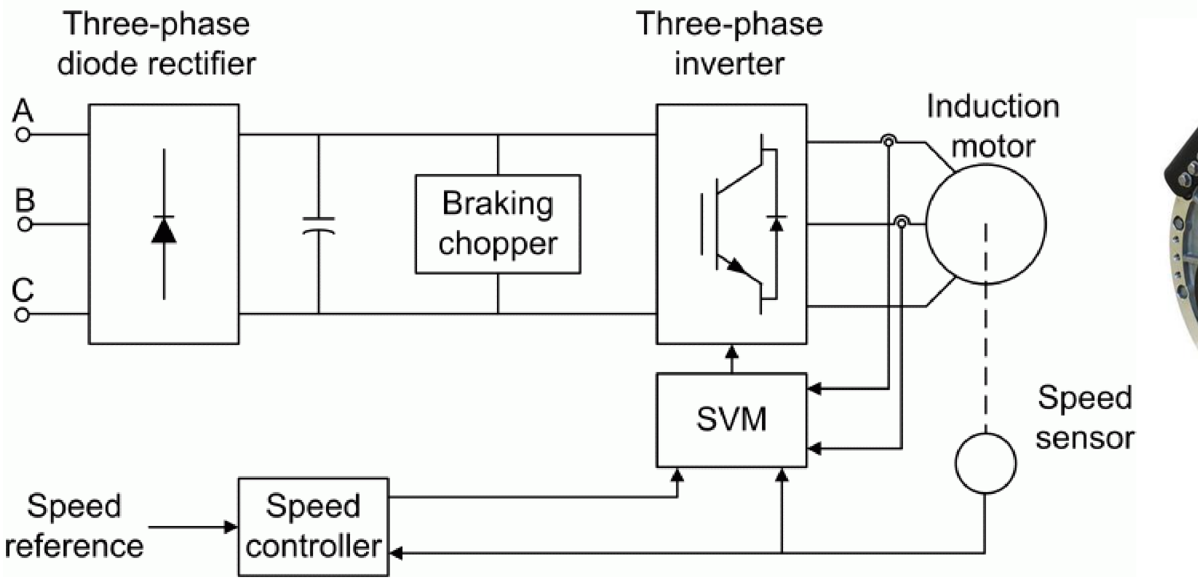
Speed Control by Changing the Rotor Resistance

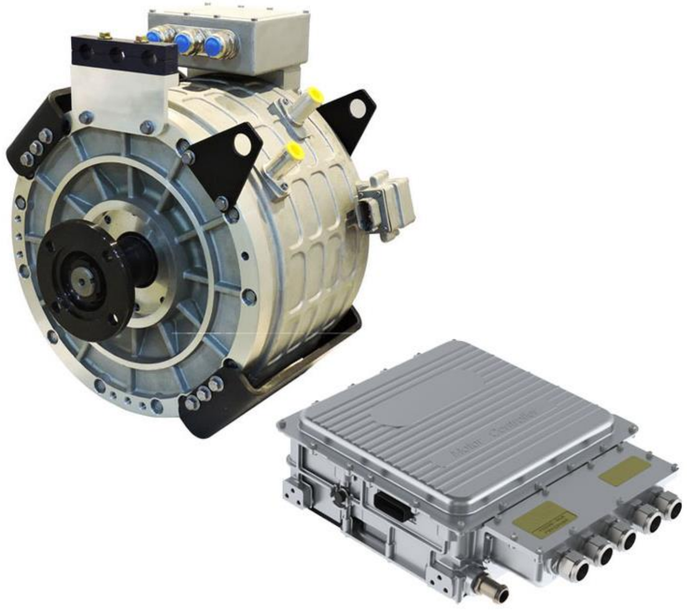
- In wound-rotor induction motors, it is possible to change the shape of the torque- speed curve by inserting extra resistances into the rotor circuit of the machine.
- This method of speed control is mostly of historical interest, since very few wound-rotor induction motors are built anymore.

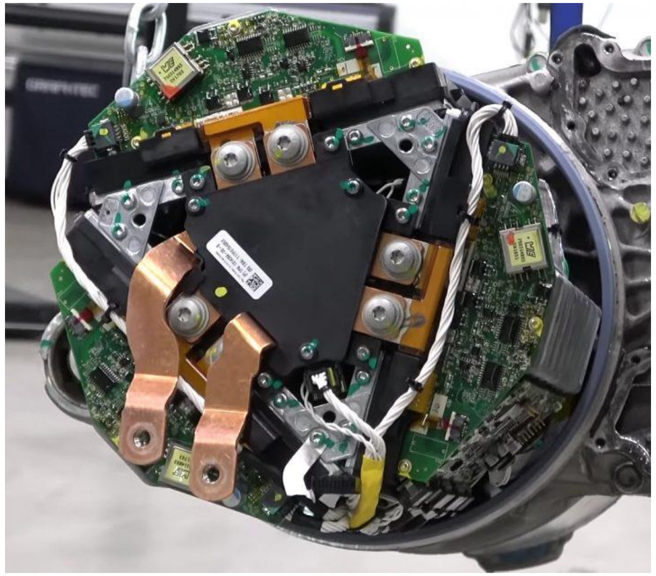


Solid-State Drives (Advanced Power Electronics)

- The method of choice today for induction motor speed control is the solid-state variable-frequency induction motor drive. The method is much more flexible and efficient due to the popular of advanced power electronics.

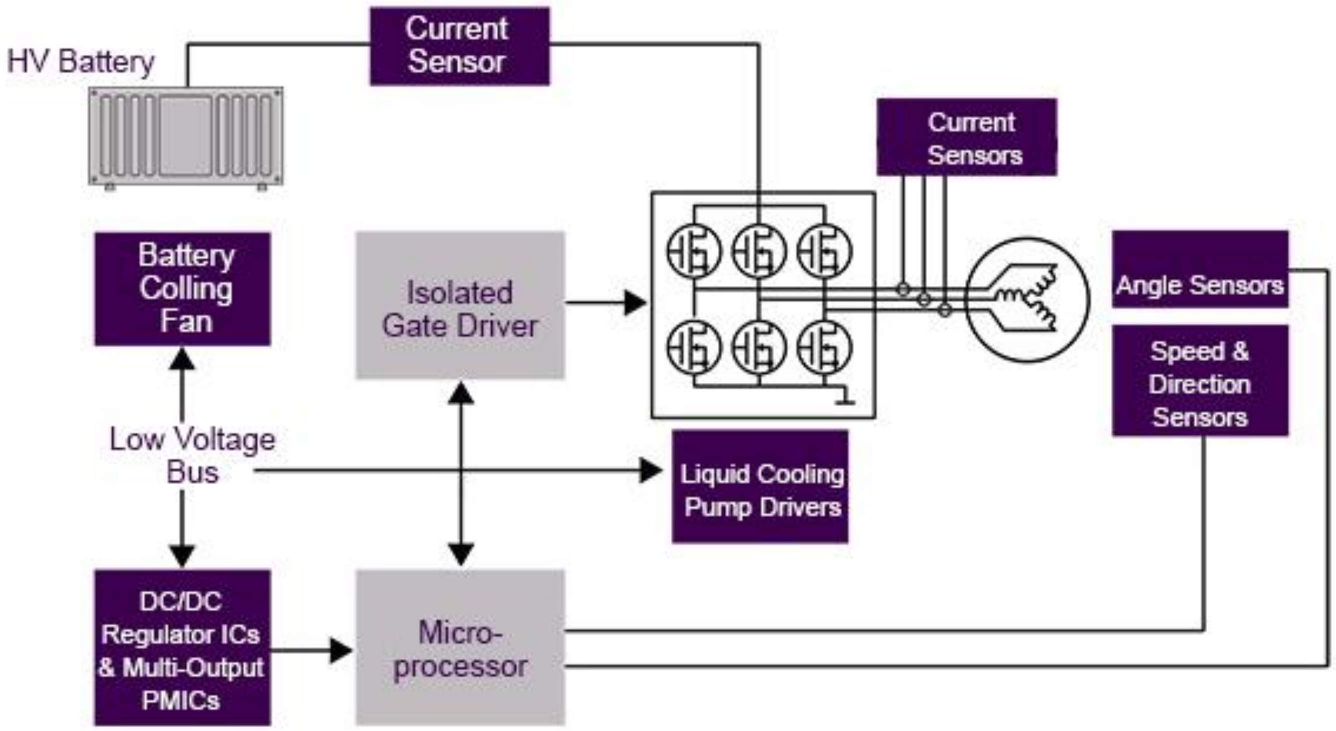


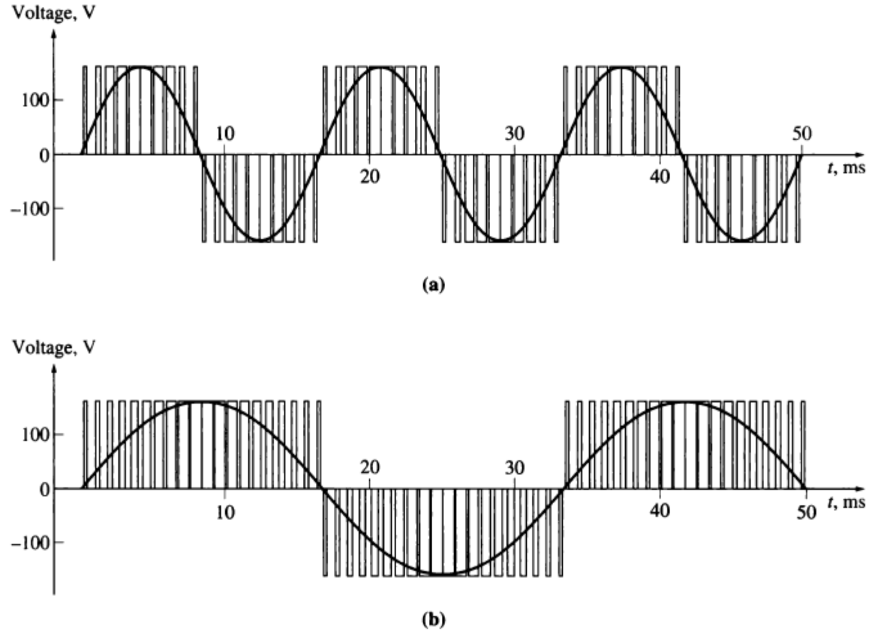


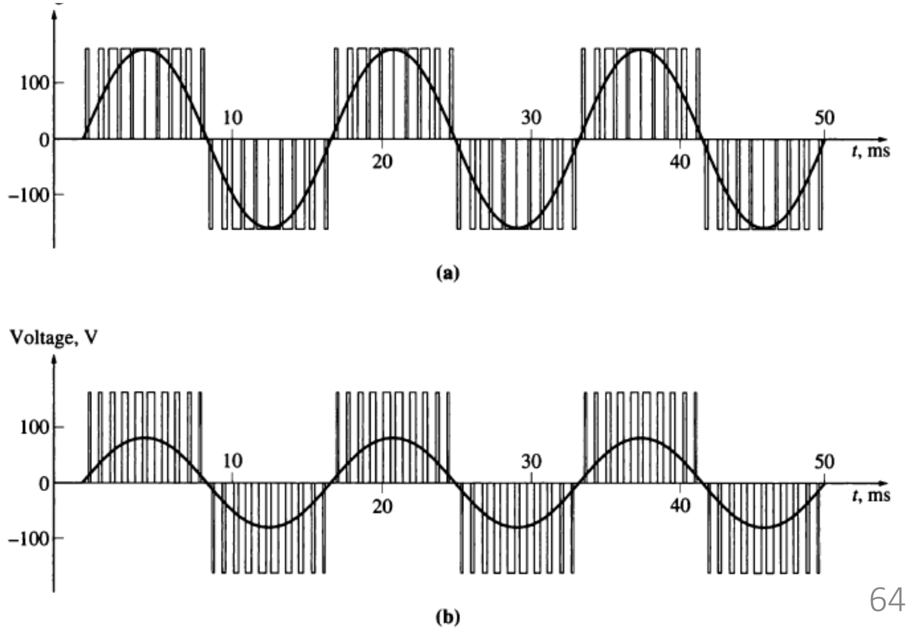


Solid-State Drives (Advanced Power Electronics)

- The output from this drive is a three-phase set of voltages whose frequency can be varied from 0 up to 120 Hz and whose voltage can be varied from 0 V up to the rated voltage of the motor.
- PWM method – high frequency drive low frequency.







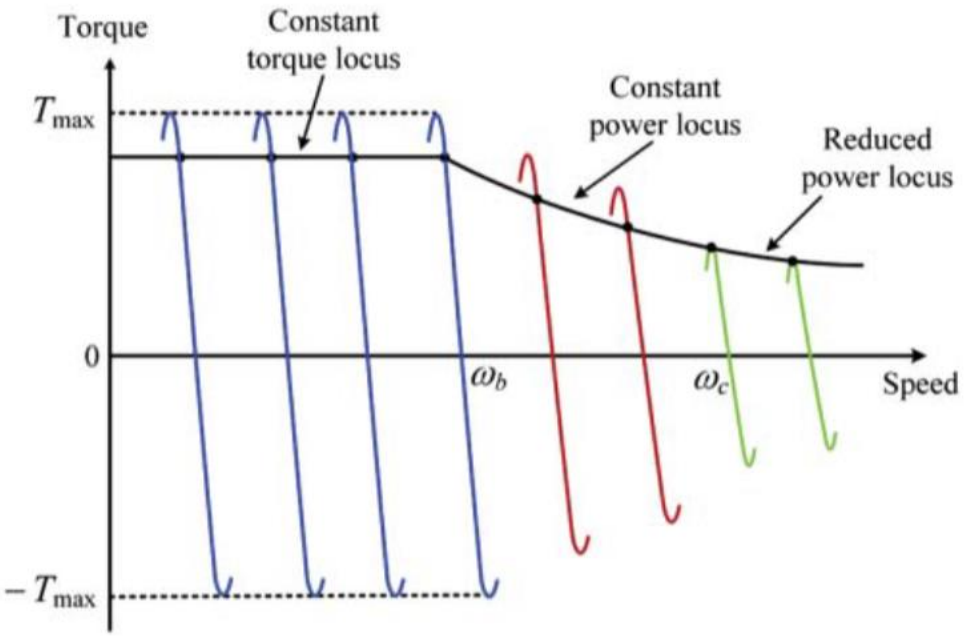
Variable-Voltage and Variable-Frequency Speed Control

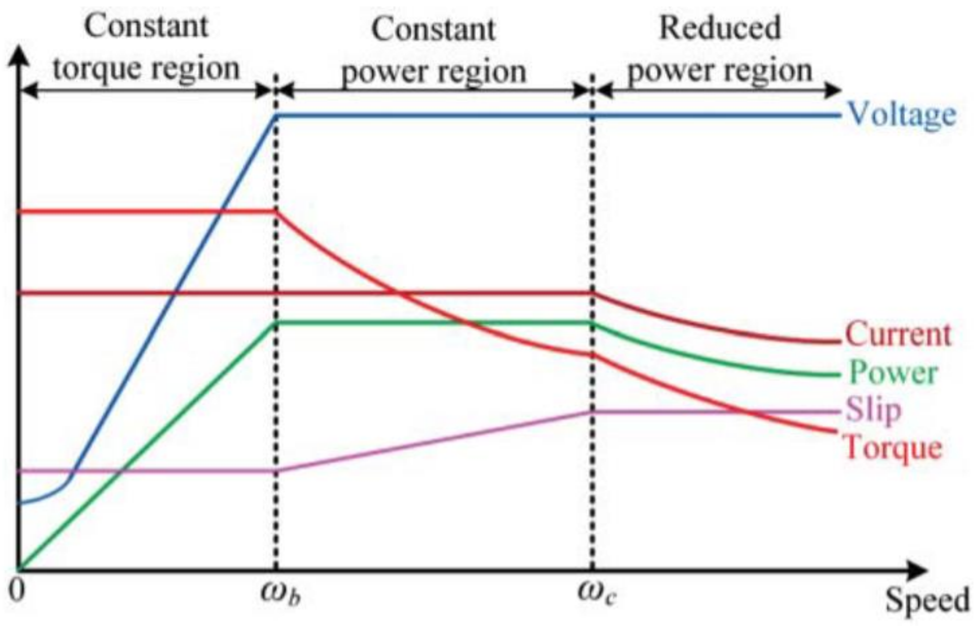
- VVVF control has been widely adopted for speed control of induction drives. It is based on constant volts/hertz control for frequencies below the rated speed/frequency, and variable-frequency control with constant rated voltage for speed/frequencies beyond the rated frequency.
- For very low frequencies, voltage boosting is applied to compensate the difference between the applied voltage and induced EMF (Dubey, 1989).
- Because the measurement of the back EMF is very difficult, the applied voltage is generally adopted to approximate the back EMF. Thus, the desired constant E⁄f strategy is approximated by the constant V⁄f strategy for most operating frequencies, except under low frequencies for low-speed operation.
- At low speeds, the stator impedance drop becomes appreciable so that the applied voltage can no longer be valid to approximate the back EMF. Thus, a boosting voltage is normally required to compensate for the stator impedance drop for low-speed operation using the constant V⁄f strategy.

*Only need to know the concept of VVVF control.

Variable-Voltage and Variable-Frequency Speed Control

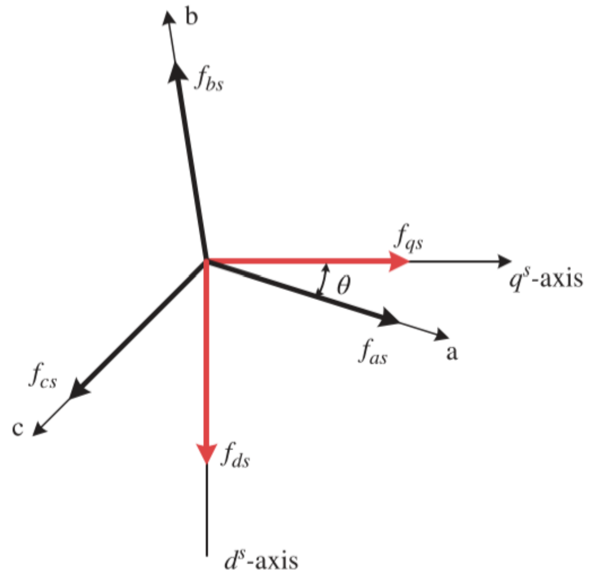
- It can be observed that there are three operating regions. The first region is called the constant-torque region in which the motor can deliver its rated torque for speeds below the rated speed (normally called the base speed ω_b).
- In the second region, called the constant-power region, the slip is increased gradually to the maximum value so that the stator current remains constant and the motor can maintain its rated power capability.



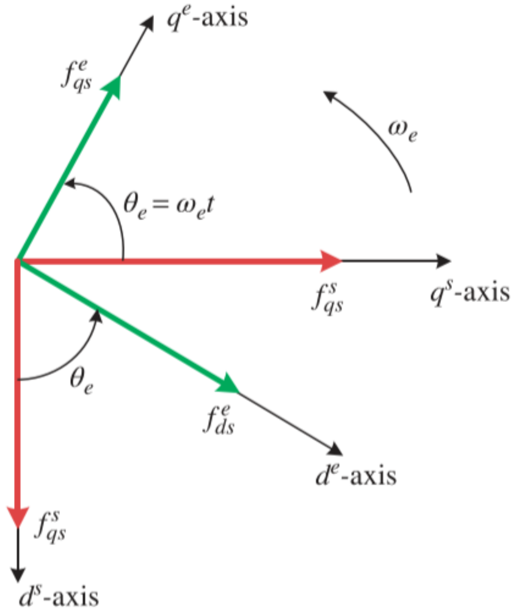


Field-Oriented Control (FOC)

- In order to improve the dynamic performance of induction motor drives, FOC is preferred to VVVF control. By using FOC, the mathematical model of the induction motor is first transformed from the stationary *a–b–c* frame to the stationary *ds–qs* frame (direct-quadrature), then to the synchronously rotating *de–qe* frame with speed ω_e. Thus, at steady state, all sinusoidal variables in the stationary frame such as the stator voltage vs, stator current is, stator flux linkage λ_s, rotor voltage v_r, rotor current i_r, and rotor flux linkage λ_r can be represented by DC quantities in a synchronously rotating.



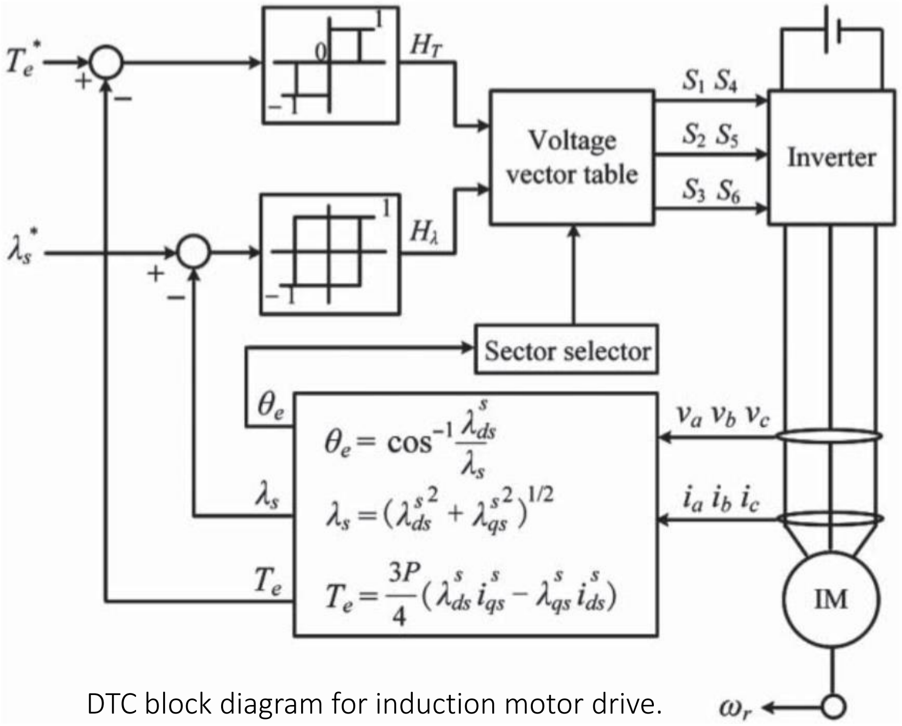
Stationary a–b–c frame to stationary $d_s - q_s$ frame



Stationary $d_s - q_s$ frame to synchronously rotating $d_e - q_e$ frame

Direct Torque Control (DTC)

- The DTC is an advanced scalar control scheme that can offer comparable performance as the FOC for the induction motor drive. This scheme is to directly control the stator flux linkage and the torque by properly selecting the switching modes of the voltage-fed PWM inverter. The selection is made to restrict the torque and flux errors within the respective torque and flux hysteresis bands, hence to achieve the faster torque response and flexible control (Vas, 1998; Bose, 2001).



DTC block diagram for induction motor drive.

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